Lumpy Durable Consumption Demand and the Limited Ammunition of Monetary Policy

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Abstract

In a fixed-cost model of durable consumption demand, we show that an important channel of monetary policy transmission is to prompt households to accelerate the timing of their adjustments. This yields an intertemporal trade-off in aggregate demand as encouraging households to adjust today leaves fewer households acquiring durables going forward. Interest rates must be kept low to support demand going forward, so accommodative monetary policy today reduces the natural real rate of interest ($r^*$) in the future. We show that this mechanism is quantitatively important in explaining the persistently low level of $r^*$ during and after the Great Recession. The decline in $r^*$ is amplified by the weak power of forward guidance relative to standard models. Our results show that monetary stimulus erodes future policy space and thereby limits the monetary ammunition available in the future.

JEL Classification: E21, E43, E52

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1 Introduction

Expenditure on durable consumption goods is a particularly interest-rate-sensitive and volatile component of aggregate demand. At the micro-level these expenditures are infrequent and lumpy. Do the micro-foundations of durable goods demand affect the monetary transmission mechanism? We explore this question using a heterogeneous-agent New Keynesian model with fixed costs for durable adjustment. Our key result is that monetary policy shifts aggregate demand intertemporally: lower interest rates accelerate durable purchases, increasing aggregate demand today at the expense of lower aggregate demand in the future. We then show that this mechanism was important in persistently reducing $r^*$ during the Great Recession and the years that followed.

Our work builds on a growing literature that models aggregate demand using rich micro-foundations for household consumption that are disciplined by micro-data.\(^1\) Most of this literature focuses its attention on the determination of non-durable consumption and abstracts from consumer durables. Our interest in durable goods is motivated by the fact that they are more sensitive to monetary policy and in general more cyclical than non-durable consumption. For example, durable goods account for two thirds of the response of total consumption to an identified monetary shock (see Section 3).

To explore these issues we develop a model of the monetary transmission mechanism in which households face fixed costs in adjusting their durable positions. Households optimally follow an (S,s) policy, making lumpy durable purchases as their existing durable stock drifts down and hits an adjustment threshold. Expansionary monetary policy shifts the adjustment thresholds, accelerating adjustments by those who were close to an adjustment threshold.

Existing studies have shown evidence of intertemporal shifting of durables purchases in response to intertemporal incentives. For example, Mian and Sufi (2012) show that the Cash-for-Clunkers program created an initial burst of auto sales that was subsequently reversed. Baker et al. (2019) find similar patterns in the response of auto sales to anticipated sales tax changes. We provide additional evidence for intertemporal shifting using Romer and Romer (2004) monetary policy shocks. Our point estimates for the responses of GDP, aggregate durable expenditure, and the extensive margins of auto sales and residential moves all show clear reversals consistent with intertemporal

For our analysis, it is important to carefully model the willingness of households to adjust the timing of their purchases in response to intertemporal price changes. House (2014) has pointed out that the timing of durable purchases is highly sensitive to intertemporal incentives in fixed-cost models. Reiter, Sveen, and Weinke (2013) show that a relatively standard \((S,s)\) model of investment demand when embedded in a New Keynesian model implies a counterfactually large investment response to monetary stimulus because so many investment adjustments are pulled forward. We compare the extent of intertemporal shifting in our model against evidence from identified monetary policy shocks and quasi-experimental evidence. We show that including two particular ingredients in the model are important to match the empirical evidence, without which the model-implied intertemporal shifting of durable demand is an order of magnitude larger than our empirical benchmarks. First, operating costs are a component of the user cost of durables that is not sensitive to interest rates, which limits the shift in the \((S,s)\) adjustment thresholds. Second, match-quality shocks introduce inframarginal adjustments, which reduces the mass of households near the adjustment thresholds. We use micro-data on durable adjustments to estimate the frequency of match-quality shocks.

The key mechanism in our analysis is as follows. After monetary policy induces a household to accelerate the purchase of a durable good, that household will no longer be close to the margin of adjustment in the near future. Therefore, absent additional stimulus, future durable demand will be below normal. There is a sense in which monetary policy borrows demand from the future. To compensate for the weakness in aggregate demand going forward, the central bank must keep real interest rates low. That is, monetary policy stimulus has a side effect of reducing the real natural rate of interest \(r^*\) in subsequent periods. To the extent that monetary policy space is limited by an effective lower bound (ELB), our results show that monetary stimulus today has the effect of reducing policy space in the future.

We use our model to quantify how the massive monetary stimulus provided by the Federal Reserve in the Great Recession affected the evolution of \(r^*\). We find that the natural rate of interest measured as a two-year yield was 3.3 percentage points lower in 2013 as a result of the monetary stimulus provided between 2007 and 2012. Thus, pulling forward aggregate demand through monetary accommodation explains more than half of the 5.8 percentage point decline
in $r^*$ between 2007 and 2013. Moreover, the effects of the recession and associated monetary accommodation on $r^*$ are very persistent—in 2018 we find that $r^*$ was still 1 percentage point below steady state due to the shocks that occurred more than 5 years earlier. We then show that these cyclical forces can account for much of the movement in the 5-year TIPS yield over the last decade.

The interaction between monetary policy and $r^*$ that we describe is very different from the prevailing neo-Wicksellian view (Woodford, 2003) that $r^*$ is largely exogenous to monetary policy and the central bank aims to manipulate the policy rate to track $r^*$.\(^2\) In contrast, we argue that monetary policy itself has a strong effect on shaping the evolution of $r^*$ through the intertemporal shifting of aggregate demand by accelerating durable purchases. This dimension of monetary policy has not previously been explored or quantified. Contemporaneous work by Mian, Straub, and Sufi (2019) argues that intertemporal shifting of demand can also occur following monetary accommodation due to the accumulation of debt by households and firms.

The transmission of monetary policy through the extensive margin of durable adjustment also implies that forward guidance is less powerful than in standard monetary models. The opportunity cost of accelerating the adjustment is the foregone interest over the time period that the adjustment is brought forward. As a result, contemporaneous interest rates are especially relevant for the timing decision, whereas forward rates are a much less powerful in shifting aggregate demand. Standard monetary models predict implausibly large output responses to forward guidance (Del Negro, Giannoni, and Patterson, 2015; Carlstrom, Fuerst, and Paustian, 2015). In response to this “forward guidance puzzle,” one strand of literature has developed alternative models in which non-durable consumption is less forward looking (McKay, Nakamura, and Steinsson, 2016; Werning, 2015; Farhi and Werning, 2017; Gabaix, 2016; Angeletos and Lian, 2018). We show that including durable goods makes aggregate demand less sensitive to promises of lower interest rates far in the future. Our findings can thereby help resolve the forward guidance puzzle. Moreover, the limited power of forward guidance contributes to the large decline in $r^*$ during and after the Great Recession.

In recent years, the low level of interest rates has received a lot attention (Summers, 2015; \(^3\)See Woodford (2003, p. 49): “In Wicksell’s view, price stability depended on keeping the interest rate controlled by the central bank in line with the natural rate determined by real factors (such as the marginal product of capital). [...] Wicksell’s approach is a particularly congenial one for thinking about our present circumstances [...]”
Laubach and Williams, 2016). These low rates are generally thought to reflect low-frequency secular phenomena such as demographic changes, the slow-down in productivity growth and the rise in income inequality (Eggertsson et al., 2019; Auclert and Rognlie, 2018; Straub, 2018). Our results, however, demonstrate that cyclical forces have large and very persistent effects on the natural rate of interest, and that these forces have contributed substantially to the low interest rates over the last decade.

A recent strand of literature has analyzed how past interest rates affect the power of monetary policy (Berger, Milbradt, Tourre, and Vavra, 2018; Eichenbaum, Rebelo, and Wong, 2018). These papers argue that the prevalence of fixed-rate mortgages in the United States leads to less refinancing activity in response to an interest rate reduction if past interest rates were low because the interest rates on existing mortgages are already low. The intertemporal shifting of demand that we describe is conceptually different from this mechanism: we argue that demand is weak going forward not that policy is less effective. Intertemporal shifting is not likely in the context of refinancing because a mortgagor that refinances to a lower fixed-rate mortgage will continue to enjoy higher disposable income on an ongoing basis and need not reduce their consumption in the future.

Our analysis uses two technical innovations building on the sequence-space approach to solving heterogeneous agent models described by Auclert, Bardóczy, Rognlie, and Straub (2019). First, we show how to implement the Kalman filter to recover the shocks that generated the aggregate time series data using only impulse response functions and not relying on a state space representation of the model. Second, we demonstrate how \( r^* \) can be immediately calculated from the impulse response functions of the model without solving an auxiliary flexible-price model as is typically done in DSGE models. Both techniques reduce the computational burden of our analysis.

Our contribution to the literature on the role of durable goods in the monetary transmission mechanism is to show and quantify the importance of the intertemporal trade-off in aggregate demand and the diminished power of forward guidance. Previous contributions have explored other roles of durables in the monetary transmission mechanism. Our partial-equilibrium household decision problem builds on Berger and Vavra (2015) who document state dependent effects of interest rate changes in an (S,s) model of durables. Barsky, House, and Kimball (2007) show that monetary non-neutrality is primarily determined by nominal rigidities in the durable sector,
and Barsky, House, Boehm, and Kimball (2016) show that this feature is important for optimal monetary policy. Sterk and Tenreyro (2018) build an OLG model, in which redistribution away from current generations due to inflationary monetary policy lowers real interest rates and creates a boom in durable spending. Kaplan, Moll, and Violante (2018) focus on the role of illiquid assets in raising the MPC and show this increases the importance of indirect effects in monetary transmission to non-durable consumption. Luetticke (2019) explores how monetary policy affects household demand for liquidity with consequences for aggregate investment in a model in which capital holdings adjust solely on the intensive margin.

The paper is organized as follows: Section 2 presents our model of demand for durable consumption goods; Section 3 discusses the empirical evidence for intertemporal shifting, compares the magnitude of intertemporal shifting in the model to the data, and explains the roles of match-quality shocks and operating costs in reducing intertemporal shifting; Section 4 describes the general equilibrium model with sticky wages; Section 5 documents the implications for the monetary transmission mechanism: intertemporal shifting and weaker forward guidance; Section 6 shows that these features of the monetary transmission mechanisms have important implications for the dynamics of \( r^* \) during and after the Great Recession; Section 7 concludes.

2 Model of Durable Demand

We begin with the household’s partial equilibrium decision problem, which forms the demand side of the model. A key step is to discipline households’ willingness to shift durable purchases intertemporally and thereby how much intertemporal shifting of demand is caused by monetary policy. Later we will embed this demand block into a sticky-wage monetary model.

2.1 Household’s Problem

Households consume non-durable goods, \( c \), and a service flow from durable goods, \( s \). Household \( i \in [0, 1] \) has preferences given by

\[
E_0 \int_{t=0}^{\infty} e^{-\rho t} u(c_{it}, s_{it}) \, dt.
\]  

(1)

The service flow from durables is generated from the household’s stock of durable goods \( d_{it} \). For the most part we have \( s_{it} = d_{it} \), but we will complicate this relationship below.
Households hold a portfolio of durables and liquid assets denoted $a_{it}$. When a household with pre-existing portfolio $(a_{it}, d_{it})$ adjusts its durable stock, it reshuffles its portfolio to $(a_{it}' , d_{it}')$ subject to the payment of a fixed cost such that

$$a_{it}' + p_t d_{it}' = a_{it} + (1 - f)p_t d_{it},$$  \hspace{1cm} (2)$$

where $p_t$ is the relative price of durable goods in terms of non-durable goods, and $fp_t d_{it}$ is a fixed cost proportional to the value of the durable stock. Liquid savings pay a safe real interest rate $r_t$. The household is able to borrow against the value of the durable stock up to a loan-to-value (LTV) limit $\lambda$

$$a_{it} \geq -\lambda(1 - f)p_t d_{it}.$$  \hspace{1cm} (3)$$

Borrowers pay real interest rate $r_t + r_t^b$, where $r_t^b$ is an exogenous borrowing spread.

The stock of durables depreciates at rate $\delta$. Following Bachmann et al. (2013), a fraction $\chi$ of depreciation must be paid immediately in the form of maintenance expenditures. This maintenance reduces the drift rate of the durable stock so we have

$$\dot{d}_{it} = -(1 - \chi)\delta d_{it},$$  \hspace{1cm} (4)$$

where a dot over a variable indicates a time derivative. The household must also pay a flow cost of operating the durable stock equal to $\nu d_{it}$. Broadly speaking these operating costs reflect expenditures such as fuel, utilities, and taxes.\footnote{We assume that operating costs do not scale with the relative durable price (in contrast to maintenance costs), but our results are essentially identical if we operating costs are proportional to the relative price.}

When a household does not adjust its durable stock, its liquid assets evolve according to

$$\dot{a}_{it} = r_t a_{it} + r_t^b a_{it} I_{\{a_{it} < 0\}} - c_{it} + y_{it} - (\chi \delta p_t + \nu)d_{it}.$$  \hspace{1cm} (5)$$

Household income, $y_{it}$, is given by

$$y_{it} = Y_t z_{it}$$  \hspace{1cm} (6)$$

where $Y_t$ is aggregate income and $z_{it}$ is the household’s idiosyncratic income share, which we later interpret as idiosyncratic labor productivity. $\ln z_{it}$ follows the Ornstein-Uhlenbeck process

$$d \ln z_{it} = \rho_z \ln z_{it} dt + \sigma_z dW_{it} + (1 - \rho_z) \ln \bar{z} dt,$$  \hspace{1cm} (7)$$
where \( dW_{it} \) is a standard Brownian motion, \( \rho_z < 0 \) controls the degree of mean reversion of the income process, \( \sigma_z \) determines the variance of the income process, and \( \tilde{z} \) is a constant such that \( \int z_{it} \, di = 1 \).

We allow for the possibility that households may occasionally adjust their durables because their existing durables are no longer a good match for them. These match-quality shocks are meant to capture unmodeled life events that leave the household wanting to adjust for reasons other than income fluctuations and depreciation. For example, a job offer in a distant city may prompt the household to move houses. Or a growing family may require a larger car. We assume that a household is in a good match when it adjusts its durables, but over time the match quality can break down according to a Poisson process with intensity \( \theta \). Specifically, there is a state \( q_{it} \) that takes a value 1 when the household adjusts its durables and drops to zero with intensity \( \theta \). The service flow is

\[
s_{it} = q_{it} d_{it}. \tag{8}
\]

In equilibrium, households with bad matches will adjust their durable stocks immediately. These match-quality shocks are therefore a source of inframarginal adjustments.

In summary, the household maximizes (1) subject to (2), (3), (4), (5), (6), (7), and (8), taking as given exogenous paths for \( Y_t, r_t, \) and \( r_t^b \).

### 2.2 Distribution and Aggregate Quantities

We use the policy functions from the household’s problem and the distribution of idiosyncratic state variables to construct aggregate quantities for a population of households. The individual state variables are the distance from the borrowing limit, \( k = a + \lambda d \), the durable stock \( d \), and idiosyncratic productivity \( z \). The distribution over these variables is denoted \( \Psi_t(k, d, z) \). In steady state, the exogenous variables are constant, \( r_t = \bar{r}, r_t^b = \bar{r}^b, Y_t = \bar{Y}, p_t = \bar{p} \), and the steady state distribution over individual states is stationary and denoted by \( \bar{\Psi}(k, d, z) \).

Aggregate nondurable expenditure is the sum of nondurable consumption and operating costs,

\[
N_t = \int c_t(k, d, z) \, d\Phi_t(k, d, z) + \nu \int d \, d\Phi_t(k, d, z),
\]

where \( c_t(k, d, z) \) is the policy function for nondurable consumption, which varies with time as the policy rule depends on the current and future values of the aggregate variables \( Y_t, r_t, r_t^b \), and \( p_t \).
Aggregate durable expenditure is the sum of net durable expenditures from adjustment, including the fixed costs of adjustment, and maintenance costs:

\[ X_t = \int \lim_{dt \to 0} \frac{\text{prob}_{t,t+dt}(k,d,z)}{dt} \left( d_t^*(k,d,z) - (1-f)d \right) d\Phi_t(k,d,z) + \chi \int d\Phi_t(k,d,z) \]

where \( \text{prob}_{t,t+dt}(k,d,z) \) is the probability that a household with individual state variables \((k,d,z)\) will make an adjustment between \(t\) and \(t+dt\), and \(d_t^*(k,d,z)\) is the optimal durable stock conditional on adjusting. Since we integrate over changes in durable stocks at the household level, \(X_t\) reflects purchases of durables net of sales of durables. Our definition of \(X_t\) is therefore consistent with the construction of durable expenditure in the national accounts, in which transactions of used durables across households are netted out.

### 2.3 Calibration of the Household Problem

We set

\[ u(c,s) = \left[ (1-\psi) \frac{1}{\xi} \frac{\xi-1}{\xi} + \psi \frac{1}{\xi} \frac{\xi-1}{\xi} \right]^{-\frac{\xi(1-\sigma)}{\xi-1}} - 1 \]

\(\xi\) is the elasticity of substitution between durables and non-durables. There is a range of estimates for this elasticity. Many housing and durable models choose an elasticity of 1, which is consistent with estimates in Ogaki and Reinhart (1998) and the near-constant expenditure share of housing in the NIPA (Davis and Ortalo-Magné, 2011). However, housing and rent expenditure shares have trended up in the AHS, Census, and CEX survey data (Albouy et al., 2016), which is more in line with an elasticity below one. Further, studies allowing for non-homothetic preferences tend to estimate below-unitary elasticities (Pakoš, 2011; Davidoff and Yoshida, 2013; Albouy et al., 2016). We choose an elasticity of \(\xi = 0.5\), which is at the lower end of these estimates. Choosing a lower value is conservative for intertemporal shifting in that the benefits of accelerating a durable adjustment are smaller.

We set \(\sigma = 4\) implying an IES of 1/4. This is at the lower end of the range typical in the literature. We need a low IES to match the small response of non-durable consumption to monetary policy shocks we measure in the data (see Section 3). The durable exponent \(\psi\) is set to match the average ratio of the nominal values of the total durable stock (durable goods and private residential structures) and annual non-durable consumption (non-durable goods and services excluding housing) from 1970 to 2019.
Like Berger and Vavra (2015), our calibration captures a broad notion of durables, which includes residential housing, autos, and appliances among other goods. While these goods differ in important respects, such as their depreciation rate and the probability of adjustment, they are all long-lasting and illiquid and purchases are lumpy and infrequent, features we stress in our analysis. Following this broad notion, our depreciation rate $\delta$ is the annual durable depreciation divided by the total durable stock in the BEA Fixed Asset tables, again averaged from 1970 to 2019. While 73% of the value of the total durable stock consists of residential housing, this component accounts for 23% of the total depreciation owing to the low depreciation rate of structures relative to cars and appliances. This explains why non-housing durables account for the majority (64%) of spending on durables and are thus important in the determination of aggregate demand.

The fixed cost of adjustment, in combination with the depreciation rate, is a key determinant of the probability of adjustment of the durable stock. We set the fixed cost to target a weighted average of the annual adjustment probabilities of individual durable goods. The three components of the average are the probability that a household moves to a new dwelling (15% Bachmann and Cooper, 2014), makes a significant addition or repair to their current dwelling (2.5% in the PSID), or acquires a new or used car (29.6% in the CEX). We attach a weight of 0.9 to the sum of housing moves and additions and repairs, and a weight of 0.1 to cars, based the the relative value of the housing stock and the car stock in the BEA fixed asset tables. This yields an annual adjustment probability of 0.19. Note that we cannot simply sum the probabilities of adjustments across durable goods, since this would overstate the liquidity of the households’ total durable position in our model.

Our calibration of the income process uses the estimates from Floden and Lindé (2001) converted to a continuous-time Ohrnstein-Uhlenbeck process. We set $\rho_z = \log(0.9136)$ and $\sigma_\eta = 0.2158$.

We set the steady state net real interest rate $r$ to 2.34%. This value is the average, ex post, real federal funds rate in terms of non-durables from 1970 to 2007. The borrowing spread $r^b$ is set to 1.7% based on the difference between the 30-year mortgage rate and the 10-year treasury bill rate over the same period. The discount rate $\rho$ is set to match the average liquid financial asset holdings net of mortgage and auto loans to annual GDP ratio since 1970 of 0.87. The borrowing limit is set to $\lambda = 0.8$ in line with a 20% down payment requirement.

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4 We use the same set of assets as McKay, Nakamura, and Steinsson (2016) and Guerrieri and Lorenzoni (2017): Currency and checkable deposits, time and savings deposits, MMF shares, Treasury securities, agency securities, municipal securities, corporate and foreign bonds, and mutual fund shares.
The fraction of depreciation covered by maintenance in housing is 92% based on the sum of intermediate goods and services consumed in the BEA housing output table and PCE on household maintenance. For cars, PCE on motor vehicle maintenance and repair is 47% of depreciation. We sum maintenance costs for cars and housing and divide by durable depreciation to get $\chi = 0.35$.\footnote{$\chi$ is not a convex combination of 92% and 47% because the denominator includes depreciation from durables other than cars and housing.}

Turning to operating costs, taxes on the housing sector, PCE on household utilities, and PCE on fuel oil and other fuels (excluding motor vehicle fuels) amount to 4.1% of the value of the housing stock. For cars, we find that PCE on motor vehicle fuels, lubricants, and fluids amounts to 22% of the value of the stock of vehicles. We sum the operating costs for cars and housing and divide by the total durable stock to obtain $\nu = 0.048$.

### 2.4 Estimating the Arrival Rate of Match-Quality Shocks

The intensity of the match-quality shock, $\theta$, does not have a natural data counterpart that lends itself to calibration. We therefore estimate this parameter using PSID data and a variant of the structural estimation method developed by Berger and Vavra (2015) that allows for match-quality shocks. We only provide a brief overview here, with details relegated to Appendix B.

The estimation method centers on matching the probability of a durable adjustment as a function of the “durable gap” $\omega_{it} \equiv d_{it}^* - d_{it}$, where $d_{it}^*$ is the optimal durable stock based on the current state variables. Specifically, $d^*$ is the level of durables the household would adjust to if it were forced to adjust immediately. Intuitively, in a fixed-cost model the probability of adjustment should be greater the larger is the absolute gap, since the benefit of adjusting the durable stock is larger.

Gaps are easily computed in the model, since both the optimal durable choice $d_{it}^{model}$ and the current durable stock $d_{it}^{model}$ are known. In the data, we only observe current durable holdings $d_{it}^{data}$ directly. We infer data gaps using a set of observables $Z_{it}^{data}$, and the model-implied relationship between them and the optimal durable stock, $d_{it}^{data} = F_{model}(Z_{it}^{data})$, where $F_{model}$ is the model’s mapping from the observables to $d^*$. We then minimize the distance between the hazard rate of adjustment in model and data, $h_{model}(\omega) - h_{data}(\omega)$ where $h(\omega)$ is the probability of adjusting given a gap $\omega$, as well the as the distance between the distribution of gaps, $f_{model}(\omega) - f_{data}(\omega)$ where $f(\omega)$ is the density at gap $\omega$. Like Berger and Vavra (2015), we allow for measurement error in...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Discount factor</td>
<td>0.104</td>
<td>Net Assets/GDP = 0.87</td>
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<tr>
<td>$\sigma$</td>
<td>Inverse IES</td>
<td>4</td>
<td>See Section 3</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Durable exponent</td>
<td>0.585</td>
<td>d/c ratio = 2.64</td>
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<tr>
<td>$\xi$</td>
<td>Elas of substitution</td>
<td>0.5</td>
<td>See text</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Real interest rate</td>
<td>0.023</td>
<td>Annual real FFR</td>
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<td>$r^b$</td>
<td>Borrowing spread</td>
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<td>Mortgage T-Bill spread</td>
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<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.068</td>
<td>BEA Fixed Asset</td>
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<tr>
<td>$f$</td>
<td>Fixed cost</td>
<td>0.187</td>
<td>Ann. adjustment prob = 0.19</td>
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<tr>
<td>$\theta$</td>
<td>Intensity of match-quality shocks</td>
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<td>See Section 2.4</td>
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<tr>
<td>$\chi$</td>
<td>Required maintenance share</td>
<td>0.35</td>
<td>See text</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Operating cost</td>
<td>0.048</td>
<td>See text</td>
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<tr>
<td>$\rho_z$</td>
<td>Income persistence</td>
<td>-0.090</td>
<td>Floden and Lindé (2001)</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Income st. dev.</td>
<td>0.216</td>
<td>Floden and Lindé (2001)</td>
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<tr>
<td>$\lambda$</td>
<td>Borrowing limit</td>
<td>0.8</td>
<td>20% Down payment</td>
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**Parameters of the Household’s Problem**

**General Equilibrium Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Source</th>
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<td>$G/Y$</td>
<td>Steady state govt share</td>
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<td>Convention</td>
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<tr>
<td>$\zeta$</td>
<td>Inverse durable supply elasticity</td>
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<td>$\kappa$</td>
<td>Phillips curve slope</td>
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<td>See Section 4</td>
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<td>$\rho_t$</td>
<td>Real rate persistence</td>
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<td>Estimated over 1991-2007</td>
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<td>$\phi_{\pi}$</td>
<td>Real rate response to inflation</td>
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<td>Estimated over 1991-2007</td>
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<td>$\phi_y$</td>
<td>Real rate response to output gap</td>
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<td>Estimated over 1991-2007</td>
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<td>$\rho_g$</td>
<td>Non-household demand persistence</td>
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<td>See text</td>
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<td>$\rho_{r^b}$</td>
<td>Borrowing spread persistence</td>
<td>-0.63</td>
<td>Estimated over 1991-2007</td>
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</table>
Figure 1: Hazard of adjustment conditional on the durable gap $\omega = d^* - d$, where $d^*$ is the optimal durable choice conditional on adjusting and $d$ the initial durable stock. Shaded areas are 95% confidence bands.

The arrival rate of the match-quality shock, $\theta$, is primarily identified by the hazard of adjustment for small gaps. Intuitively, at small gaps the household should be relatively far from an adjustment threshold due to the fixed cost, whereas adjustments at large gaps likely reflect that the household crossed an adjustment threshold. Most of the adjustments at small gaps are therefore attributed to the match-quality shock. As we vary $\theta$, we re-calibrate the discount rate $\rho$, the fixed cost $f$, and the preference for durables $\psi$ to match the same targets for net assets, the probability of adjustment, and the durable-stock-to-non-durable-consumption ratio.

Our set of observables from the PSID, $Z_{it}^{data}$, are net liquid assets $a_{it}$, the value of the durable stock $d_{it}$, and annualized consumption expenditures over the following wave $\bar{c}_{i,t,t+2}$. We use direct information from the PSID on durable adjustments. For housing we use the answer to whether the household has moved along with the recorded date of moving, and whether the household made a major addition or repair to the home. Similarly, if any car acquisition occurred after the previous wave, then we record this as a car adjustment. Just like our total adjustment frequency, we weight a housing adjustment by 0.9 and car adjustments by 0.1.

Figure 1 plots the model- and data-implied hazards of adjustment conditional on the durable gap at the optimal parameter estimate, $\theta = 0.155$. The bootstrapped 95% confidence band is
[0.154,0.156], based on sampling households from the PSID with replacement. The model accounts well for the upward-sloping hazard and explains 79% of the variation in the hazard rate. Note that the probability of adjustment is substantial in both model and data for even small gaps, suggesting that the match-quality shock is important in matching the data.\footnote{We estimate larger adjustment probabilities at small gaps than Berger and Vavra (2015) do because we follow a different approach to identifying adjustments in the data. Berger and Vavra exclude durable adjustments smaller than 20% of the value of the durable stock in part to filter out idiosyncratic moves across location.} Indeed, our estimate implies that 74% of all adjustments are due to the match-quality process. This split between match-quality shocks and adjustments due to depreciation or income fluctuations roughly coincides with the stated reasons for moving house in the March CPS Supplement (see Appendix Table 5).

3 The Response of Durable Demand to Monetary Policy

In fixed cost models, the timing of durable adjustments are very sensitive to intertemporal incentives (Reiter et al., 2013; House, 2014). As a result, a key transmission mechanism of monetary policy is to shift the timing of purchases. In this section we show that the strength of this transmission channel in our model is consistent with several empirical benchmarks.

3.1 Evidence from Identified Monetary Shocks

We estimate the response of total durable expenditure, $x_t$, and the extensive margin of durable purchases to an identified monetary policy shock.\footnote{Appendix C details the construction of the variables used in this analysis.} To measure the extensive margin, we construct time series for fractions of the population moving residence and buying a car using micro-data from the PSID and the CEX.\footnote{We construct the annual time series for the probability of moving to a different residence using PSID data from 1969-1997 following Bachmann and Cooper (2014). Bachmann and Cooper (2014) show that the moving probability from the PSID is in line with the shorter time series from the CPS March Supplement and the AHS. For the probability of buying a car we use CEX data from 1980-2007. Appendix Figure A.1 shows that CEX car expenditures aggregate well to NIPA expenditures. See Appendices C.2 and C.3 for details.} We also estimate the response of the real interest rate in terms of non-durable goods $r_t$, aggregate income $Y_t$, and the relative durable price $p_t$, which we will use in the following subsection when comparing the model to the data.

We use monetary shocks from Romer and Romer (2004), extended by Wieland and Yang (2017), for 1969Q1-2007Q4. We estimate the impulse responses of GDP, the real interest rate, the relative price of durables, aggregate durable expenditure, auto purchases, and residential moves. For the latter three outcomes, we report cumulative responses to show the extent to which the initial effects
of stimulus are later reversed. We estimate these responses using a VAR for a subset of the variables and local projections for the others.\footnote{We estimate a VAR with 16 lags of the monetary shock, the real interest rate, log GDP, and the relative price of durables, in which the monetary shock is ordered first. The VAR also includes a time-trend and standard errors are block-bootstrapped. For cumulative durable/non-durable expenditures and the cumulative extensive margin responses we use local projections. The local projections condition on 16 lags, a time trend, and standard errors account for serial correlation. (Plagborg-Møller and Wolf, 2019) show that VARs and local projections yield the same impulse response up to the horizon of included lags (16 quarters in our case). We prefer the VAR for the response of inputs to the household problem as it generates smoother impulse responses at long horizons than we obtain with local projections. We prefer the local projection for the cumulative responses since these variables are non-stationary.}

The top-left panel of Figure 2 displays an economically sizable and statistically significant decline in the annualized, ex post, real federal funds rate in terms of non-durables given a one-standard deviation expansionary monetary policy shock.\footnote{We normalize the Romer shock to yield a 25 basis point decline in the real interest rate on impact.} In the top-right panel the relative durable price displays a small increase but remains close to the steady state.\footnote{Appendix Figure A.2 shows that while the relative durable price is close to flat, both non-durable and durable prices increase with some lag following an expansionary monetary policy shock.}

The center-left panel shows that GDP increases with a lag. After 16 quarters the response of GDP turns from positive to negative, indicating that a reversal in demand is taking place. The following three graphs for cumulative real durable expenditure, the cumulative moving probability, and the cumulative car acquisition probability provide further evidence for intertemporal shifting. The cumulative durable expenditure peaks at 14\% after 15 quarters. In other words, durable expenditures are above normal for the first 15 quarters. However, durable expenditures subsequently come in below normal, as evidenced by the decline in cumulative durable expenditure. By the end of the impulse response horizon, cumulative durable expenditures have fallen to around one third of their original peak. Thus, the point estimate implies that two thirds of the increase in durable expenditures was subsequently reversed. Of course, as in the case of GDP, there is considerable uncertainty about the size of this reversal.

The bottom panels for the cumulative moving probability and the cumulative car acquisition provide suggestive evidence that this reversal is driven by the extensive margin of adjustment. In particular, for both adjustment probabilities we see a peak in the cumulative impulse response function at around 12-16 quarters, followed by a near-complete reversal. This suggests that households accelerate durable purchases following an expansionary monetary shock, and that these adjustments are subsequently missing.
Figure 2: Impulse response function of the real interest rate in terms of non-durables (top-left panel), the relative durable price (top-right), real GDP (center-left), cumulative real durable expenditure (center-right), cumulative probability of moving house (bottom-left), and cumulative probability of buying a car (bottom-right), to a one-standard-deviation Romer and Romer monetary policy shock.
3.2 Evaluating the Model

We now benchmark how willing households are to shift the timing of their durable adjustments in the model versus the data. We make this comparison based on the peak cumulative response in durable expenditure and the extensive margin of adjustment. Intuitively, the peak measures how many adjustment have been pulled forward due to expansionary monetary policy.

To measure the peak cumulative durable and extensive margin responses to monetary policy shocks in the model we feed the mean impulse response function for the real interest rate, \( r_t \), real GDP, \( Y_t \), and the relative durable price, \( p_t \), into the household decision problem of Section 2 starting in steady state. We assume that these paths come as a surprise at \( t = 0 \), but are subsequently known with certainty, and return to steady state after 16 quarters.\(^{12}\) We conduct this simulation for several versions of our model to show the contributions of operating costs and match-quality shocks.

We start with a standard fixed-cost model with idiosyncratic risk and borrowing constraints. Next, we add operating costs and finally match-quality shocks to arrive at our full model. Each of the models with fixed costs is calibrated to match the same durable-to-non-durable-consumption ratio, steady state adjustment probability, and net-liquid-assets-to-GDP ratio.\(^ {13}\)

\(^{12}\)We found that incorporating the uncertainty around the estimated price and income responses into our model simulation made little difference to the dynamics of durable demand.

\(^{13}\)In the fixed-cost model the discount rate, durable preference, and fixed cost are \( \rho = 0.085, \psi = 0.463, f = 0.018 \). In the model with operating costs they are \( \rho = 0.086, \psi = 0.545, f = 0.011 \).
The left panel of Figure 3 plots the model-implied cumulative durable expenditures against the data. In the fixed-cost model model, cumulative real durable expenditures peaks at 104%, 8 times more than their peak response in the data. This result reflects the sensitivity of the extensive margin and suggests that in this standard fixed-cost model households are too willing to adjust the timing of their durable purchases, so that there is too much intertemporal shifting relative to the data. Including operating costs reduces the peak response to 57%, which halves the discrepancy with the data to a factor of 4. With the addition of match-quality shocks, the full model comes even closer to matching the interest-elasticity of durable demand in the data. The peak real durable response is 20% compared to 14% in the data. The full model response thus falls within 95% confidence interval, [4,24%]. Thus, the willingness of households to shift the timing of durable adjustments in the full model appears to be in line with the data. Finally, note that all models predict that the increase in durable expenditures is subsequently reversed.

In the right panel of Figure 3 we show that the fixed-cost model also has difficulty matching the cumulative expenditure on non-durables. Households are spending so much on durables that they end up cutting back on non-durables. By contrast, the models including operating costs provide a good quantitative match. This figure motivated our choice of a relatively low intertemporal elasticity of substitution in Section 2. Note that cumulative non-durable consumption excluding rent increases by 1.7%, ten times less than the durable response. As durable expenditure makes up 22.9% of total consumption excluding rent, the durable component accounts for 70% of the total increase of consumption expenditure following a monetary shock.

We now show that the willingness of households to shift the timing of their durable adjustments in our full model also aligns well with the observed extensive margin responses for cars and housing. We consider two different calibrations to tailor the model to cars and housing, respectively. The primary difference is in the calibrations is the depreciation rate. As above, we simulate the  

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14 Models with high interest elasticity of durable demand will show very little fluctuation in the durables real interest rate in equilibrium (Barsky et al., 2007). Appendix Figure A.3 plots the impulse response for the durables real interest rate, which displays an economically sizable, persistent, and statistically significant decline following an expansionary monetary policy shock.

15 Housing structures depreciate at a much slower rate, 2% per year, while cars depreciate at a much higher rate, 20% per year, than the value-weighted durable stock. The probability of adjustment is also higher for cars (7.4% quarterly) than for housing (15% annually), and households own more housing wealth \( \frac{d}{c} = 1.92 \) than car wealth \( \frac{d}{c} = 0.201 \). We recalibrate the discount rate \( \rho \), the fixed cost \( f \), and the durable exponent \( \psi \) to match these targets, as well as a net-liquid-asset-to-GDP ratio of 0.92 for housing and 1.31 for cars. When we include match-quality shocks, 74% of all adjustments will come from the match-quality process, which is the same fraction as in our estimated model for all durables. This requires \( \theta = 0.12 \) for housing and \( \theta = 0.22 \) for cars. We only subtract the collateralized loans for the durable we calibrate to. We also allow for a higher borrowing spread \( r^b = 0.03 \) in the car.
impulse response for the extensive margin by feeding the empirical impulse responses of $Y_t$, $r_t$, and $p_t$ into the model.\textsuperscript{16} Table 2 has the results. Starting with the fixed-cost model, the extensive margin of housing adjustments responds 16 times more in the model than in the data while the extensive margin of car purchases is 4 times larger than in the data. The low depreciation rate of structures accentuates the distance between model and data relative to our baseline, whereas the high depreciation rate of cars attenuates it. Including operating costs dampens the extensive margin response for housing, but has little effect on the response of car adjustments. When the depreciation rate is low and there are no operating costs, the interest rate dominates the user cost of durables, which explains why the operating cost is so important for the housing model. Finally, in our full model including match-quality shocks, the magnitude of the extensive margin response is close to the empirical point estimates and well within the 95% confidence interval for both cars and housing.

Table 2: Peak Cumulative Extensive Margin Response (percent change in frequency of adjustment)

<table>
<thead>
<tr>
<th></th>
<th>Housing</th>
<th>Cars</th>
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</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[95% CI]</td>
<td>10.9</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>[2.1,20.3]</td>
<td>[-2.0,13.0]</td>
</tr>
<tr>
<td>Fixed Cost</td>
<td>170.6</td>
<td>23.1</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>40.6</td>
<td>26.1</td>
</tr>
<tr>
<td><strong>Full Model</strong></td>
<td>6.1</td>
<td>4.8</td>
</tr>
</tbody>
</table>

While the full model accords well with the empirically measured willingness to accelerate durable adjustments, households in the model do react more quickly to monetary shocks than households in the data. This is reflected in the fact that our peak cumulative expenditure or extensive margin responses occur earlier in the model than in the data. Matching these slow dynamics in the data is a well-known challenge and our model is no exception. But for our analysis it is important that the overall magnitude of intertemporal shifting of durable expenditure is consistent with the data. That quantity is key to determine how much monetary policy borrows aggregate demand from the model based on the average spread of four-year car loans with treasury bonds.

\textsuperscript{16} We use a different relative price response: for housing we use the relative price of housing to non-durable consumption, whereas for cars we use the relative price of cars. Appendix Figure A.4 shows that these relative prices behave similarly to the relative price for all durables.
future. Further, the quasi-experimental evidence in the next section suggests that intertemporal shifting can also respond quickly to intertemporal price changes.

### 3.3 Quasi-Experimental Evidence

Additional empirical evidence for intertemporal shifting of durable demand in response to a change in intertemporal prices comes from Mian and Sufi (2012) and Baker et al. (2019), who analyze the response of auto purchases to the Cash-for-Clunkers program and anticipated sales tax increases, respectively. The Cash-for-Clunkers program and anticipated sales tax increases can be interpreted as a low real interest rate, as the after-tax durable price is expected to increase both when Cash-for-Clunkers ends and when the sales tax rises. Both of these studies show a short-lived increase in auto purchases that subsequently reverses. In both cases, cumulative auto purchases are approximately zero shortly after the end of the stimulus.

We now show that our model is quantitatively consistent with their estimates for the peak cumulative effect. We will translate the results of these two studies into an interest elasticity of the extensive margin and compare to our model’s predictions. (Baker et al., 2019, Table 3, Col. 1) estimate a cumulative 12.7 percent increase in monthly auto sales leading up to a 1 percentage point increase in sales tax. This implies an annualized 12 percent decrease in the real interest rate for cars in the month before the tax increase so the elasticity of the extensive margin of auto sales to interest rates is about $12.7/12 = 1.1$. Our conversion of the estimates for the Cash-for-Clunkers program in Mian and Sufi (2012) yields an elasticity in the range of 3.2 to 3.7.\(^{17}\)

In our model calibrated to cars, we calculate the elasticity based on the impact effect of a one month drop in the real interest in durables. As shown in Table 3, our model produces an elasticity of 3.8, which is close to those derived from Mian and Sufi but somewhat larger than in Baker et al.. The table shows that operating costs and match-quality shocks are crucial in matching this evidence.

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\(^{17}\)Mian and Sufi (2012) argue that cross-city variation in Cash for Clunkers explains between 340k and 398k of additional autos sold. New vehicle sales in April and May were on average 833,000. Used vehicle sales were 36.5m and 35.5m in 2008 and 2009, implying an average monthly sales volume of 3m. Total baseline vehicle sales are then 3.833m. The increase in vehicle sales estimated by Mian and Sufi then corresponds to $340k/3.833m − 1 = 8.9\%$ to $398k/3.833m − 1 = 10.4\%$ rise. Total expenditure on Cash for Clunkers was $3bn, and the vehicle stock in 2008 was worth $1,279.4bn at replacement cost. This translates into a $3/1279.4 = 0.23\%$ percent reduction in the replacement price, or 2.8\% expressed as an annual rate. Therefore, the elasticity ranges from $8.9/2.8=3.2$ to $10.4/2.8=3.7$. 

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Table 3: Implied Interest Rate Elasticity for Car Purchases

<p>| | |</p>
<table>
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<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
</tr>
<tr>
<td>Baker et al. (2019)</td>
<td>1.1</td>
</tr>
<tr>
<td>Mian and Sufi (2012)</td>
<td>3.2-3.7</td>
</tr>
<tr>
<td>Model</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost</td>
<td>47.7</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>35.2</td>
</tr>
<tr>
<td>Full Model</td>
<td>3.8</td>
</tr>
</tbody>
</table>

3.4 The Roles of Match-Quality Shocks and Operating Costs

Why do operating costs and match-quality shocks reduce the number of durable adjustments that are pulled forward when interest rates fall? Match-quality shocks are a source of inframarginal adjustments. We target a certain probability of adjustment in total and by associating more of these adjustments with the match-quality shock, fewer are attributed to households that have hit an \((S, s)\) band. Therefore including match-quality shocks means there are fewer households near the adjustment thresholds who can be induced to accelerate their adjustments by monetary policy.\(^{18}\) To demonstrate this role of match-quality shocks, Figure 4 shows the density of durable holdings in the fixed-cost model and the full model for a particular level net liquid assets and idiosyncratic income. The vertical dashed lines show the adjustment thresholds. The mass of households near the adjustment threshold is substantially smaller in the full model.

Operating costs are a component of the user cost of durables that is not sensitive to interest rates and including them stabilizes the user cost and therefore durable demand. In Figure 4, the adjustment thresholds shift to the right in response to monetary stimulus, but much less so in the full model.

\(^{18}\)The logic of how match-quality shocks affect the extensive margin response of durable demand has antecedents in the literature on price setting (see Golosov and Lucas, 2007; Midrigan, 2011; Nakamura and Steinsson, 2010; Alvarez et al., 2016).
Figure 4: Distribution of households over durable holdings. The distributions shown are conditional on net liquid assets of 1.99 and gross income of 0.93. Net liquid assets are measured as $a - \lambda d$. The adjustment thresholds are shown at their steady state positions and in response to a 2.5 p.p. cut in the interest rate for the current quarter.

4 General Equilibrium Model

So far we have focused on the household problem, which serves as the demand side of our model. We now specify the supply side and market clearing conditions. With a full general equilibrium model we can then study the monetary transmission mechanism and determine the importance of intertemporal shifting for the dynamics of $r^*$ during the Great Recession.

4.1 Labor Supply, Production, and Aggregate Supply

We adapt the standard sticky-wage environment developed by Erceg et al. (2000) to allow for uninsured idiosyncratic labor productivity. Each household $i$ supplies a continuum of differentiated labor of type $j \in [0, 1]$, with hours denoted $n_{ijt}$. We extend the household preferences with an additively separable disutility of labor supply

$$E_0 \int_{t=0}^{\infty} e^{-\rho t} \left[ u(c_{it}, s_{it}) - \bar{u}_c \int_0^1 v(n_{ijt}) \, dj \right] \, dt.$$  

(9)
where we use the modifier on the disutility of labor, $\bar{u}_{c,t} = \int_0^1 \frac{\partial u(c_{it}, s_{it})}{\partial c_{it}} \, di$, to eliminate wealth effects on labor supply. Labor supply is determined by a set of unions as described below so the household takes labor supply and labor income as given. As the disutility of labor is additively separable and labor income is outside the household’s control, the decision problem we analyzed in the previous sections is unchanged.

Final goods are produced with a technology that is linear in labor, $Y_t = Z_t L_t$, where $Z_t$ is the exogenous productivity and $L_t$ is an aggregate of labor supply given by

$$L_t = \left( \int_0^1 l_{jt} \varphi^{-1} \, dj \right)^{\frac{\varphi}{\varphi-1}},$$

where

$$l_{jt} = \int_0^1 z_{it} n_{ijt} \, di.$$

We now interpret $z_{it}$ as idiosyncratic labor productivity. In this formulation, each household faces uninsurable risk to their productivity $z_{it}$, but face the same (relative) exposure to each variety of labor $j$. The final good is produced by a representative firm. Prices are flexible and equal to nominal marginal cost: $P_t = W_t / Z_t$, where $W_t$ is the price index associated with the aggregator $L_t$. The real wage is then $W_t / P_t = Z_t$. We assume that the movements in $Z_t$ are permanent, $\ln Z_t = \epsilon_t Z_t$, so that $\epsilon_t Z_t$ has the interpretation of a permanent income shock.

The final good is used for several purposes: non-durable consumption, an input into durable production, and government consumption. Appendix E shows the market-clearing conditions.

We obtain an upward-sloping Phillips curve through sticky nominal wages. A continuum of unions set the wage, $W_{jt}$, of each type of labor. The union maximizes the equally-weighted utility of the households subject to a Rotemberg-style adjustment cost of $\frac{\Phi}{2} \bar{u}_{c,t} L_t \left( \mu_{jt} \right)^2$, where $\Phi$ is a parameter that controls the strength of the nominal rigidity and $\mu_{jt}$ is the growth rate of $W_{jt}$ such that $\ln W_{jt} = \mu_{jt} dt$. Among union workers supplying type $j$, all labor is equally rationed, $n_{ijt} = l_{jt}$. In a symmetric equilibrium, all workers supply $L_t$ units of labor and each household receives real, pre-tax income of $z_{it} Y_t$.

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19 We do so for computational convenience. The wealth effect in labor supply introduces an additional loop in finding the market-clearing prices and quantities since it creates a feedback from distribution of marginal utility to the real interest rate in the policy rule. In unreported results we have found that this wealth effect is not quantitatively important for output dynamics.
Appendix E presents the union’s problem and shows that the log-linearized symmetric equilibrium gives rise to the following Phillips curve

$$\pi_t = \rho \pi_t - \kappa \left( \frac{Y_t - \bar{Y}_t}{Y_t} \right),$$

(10)

where $\pi_t = \frac{d\ln P_t}{dt}$ and $\bar{Y}_t$ is potential output.

The durable good is produced by a unit measure of perfectly competitive firms using the production function,

$$X_t = \nu Z_t M_t^{1-\zeta} \bar{H}^{\zeta},$$

where $X_t$ is the production of durables, $M_t$ is the input of the non-durable good, and $\nu$ is a constant. The constant flow $\bar{H}$ of land is made available and sold by the government at a competitive price. $Z_t$ enters the production function here in a manner that is “land-augmenting” so that the long-run relative price of durables is unaffected by permanent TFP shocks. The first order conditions of this problem lead to a relative price of

$$p_t = (1 - \zeta)^{-1} \nu^{-\frac{1}{1-\zeta}} \left( \frac{X_t}{Z_t \bar{H}} \right)^{\frac{\zeta}{1-\zeta}},$$

(11)

where $X_t$ is aggregate durable goods production. Thus, $(1 - \zeta)/\zeta$ is the supply elasticity of the durable good.

4.2 Government

Monetary policy is governed by a standard interest rate rule,

$$\hat{r}_t = \rho_r (r_t - \bar{r}) + \phi_r \pi_t + \phi_y \frac{Y_t - \bar{Y}_t}{Y_t} + \epsilon_t,$$

(12)

where the first term captures interest rate smoothing, the second and third terms the endogenous monetary policy response, and $\epsilon_t$ is an exogenous shock.

Fiscal policy consists of a constant debt policy,

$$A_t = \int_0^1 a_{it} dt = \bar{A}.$$

We assume that the government levies taxes proportional to $z_{it}$ where the tax rate $\bar{\tau}_t$ is set to satisfy the government budget constraint so we have

$$y_{it} = (Y_t - \bar{\tau}_t) z_{it}$$

(13)
and the period-by-period government budget constraint is

\[ \bar{\tau}_t = r_t \bar{A} + G_t \]

where \( G_t \) is an exogenous level of government consumption.\(^{20}\) We assume that it follows an autoregressive process in logs, \( d \ln G_t = \rho_g \ln G_t + \epsilon_t^G. \) In our analysis, government consumption will stand in for changes in demand that originate outside the household sector and we will at times refer to “non-household demand.”

### 4.3 Calibration of the General Equilibrium Model

We set \( \nu \) so as to normalize the relative price of durables to one in steady state. We calibrate the inverse supply elasticity of durable goods to \( \frac{\zeta}{1 - \zeta} = 0.049. \) Our choice of \( \zeta \) reflects land’s share in the production of durables, which we calculate as follows. Residential investment is on average 36% of broad durable consumption expenditures (NIPA Table 1.1.5, 1969-2007). New permanent site structures account for 58% of residential investment (NIPA Table 5.4.5). Davis and Heathcote (2007) report that 11% of sales of new houses reflect the value of land. Therefore payments for new land amount to a little over 2% of the expenditure on durables. However, Davis and Heathcote (2007) also report that the existing stock of housing is paired with more valuable land and land accounts for 36% of the value of the housing stock, which is substantially larger than the 11% share in new housing. In our model, durables trade at a single price so there is no distinction between the cost of creating new durables and the value of the stock. We therefore take the mid-point of 11% and 36%, which implies that payments to land account for 5% of expenditure on durables.

An elastic supply of durable goods is consistent with the lack of a relative price response in Figure 2. An elastic supply of durable goods also finds some support from Goolsbee (1998) and House and Shapiro (2008) who present evidence on the response of capital goods prices to policies that stimulate investment demand. House and Shapiro find little evidence of a price response and argue for a high supply elasticity. Goolsbee argues for less elastic supply in general, but for the categories of goods that also serve as consumer durables (autos, computers, and furniture) he finds little price response.

The slope of the Phillips curve is 0.48. Note that the slope of the Phillips curve is expressed in

\(^{20}\)The government also raises a small amount of revenue from selling land. In steady state this amounts to 0.5% of GDP. For computational convenience we assume this revenue finances an independent stream of spending.
terms of the change in annualized inflation for a unit of the output gap per year so we would need to divide by 16 to compare to a quarterly discrete-time model, which yields a slope of $0.48/16 = 0.03$. That value is squarely in the middle of empirical estimates (Mavroeidis et al., 2014).

We estimate the monetary policy rule from 1991-2007, since there is no significant trend in the real rate over this period. This yields $\rho_r = -0.48$ (equivalent to a quarterly persistence of 0.89), $\phi_\pi = 0.43$ and $\phi_y = 0.66$. We also estimate a autoregressive process for borrowing spread over 1991-2007, which yields $\rho_{rb} = -0.63$ and is equivalent to a quarterly persistence of 0.85. We set the persistence of the non-household demand shock $\rho_g = -0.9$ equivalent to a quarterly persistence of 0.8. We deliberately choose a value at the lower end of the persistence spectrum typically estimated for demand shocks, since a more persistent shock naturally has more persistent effects on $r^*$. This is a conservative choice, since we emphasize the prolonged low levels of $r^*$ after the Great Recession.

4.4 Solving the Model

We compute perfect foresight transition paths. For a model that is linear in aggregate states and aggregate shocks, these paths are equivalent to the impulse response functions obtained from a stochastic linear rational expectations solver (see Boppart, Krusell, and Mitman, 2018).

Our analysis assumes that the economy’s dynamics are linear the aggregate states but can be non-linear in idiosyncratic states in the style of perturbation approaches to solving heterogeneous-agent models (e.g. Reiter, 2009). While our calculations assume this linearity in aggregates, we will show robustness results that give us some confidence that our results are not too sensitive to non-linearities arising from state dependence in the power of monetary policy or arising from the ELB.

To compute the steady state and the impulse response functions for the model we use the continuous-time methods described in Achdou et al. (2017). We express the model variables relative to productivity $Z_t$ and we solve for transition paths at a quarterly frequency.

21The long-run responses are $\frac{\phi_\pi}{\rho_r} = 0.90$ and $\frac{\phi_y}{\rho_r} = 1.39$. Note that our estimated rule satisfies the conventional Taylor principle since it is specified in terms of a real rate.
5 The Monetary Transmission Mechanism

The acceleration of durable purchases in response to monetary policy stimulus has two implications for the monetary transmission mechanism that we highlight. First, monetary policy shifts aggregate demand intertemporally, which means stimulus now reduces demand in the future. Second, forward guidance is less powerful than implied by standard New Keynesian models. In Section 6 we show that these two implications are quantitatively important for the dynamics of the natural rate of interest during and after the Great Recession.

5.1 An Illustration of Intertemporal Shifting

To show the intertemporal shifting effect of monetary stimulus we consider a simple experiment in which the central bank reduces the real interest rate by 1% (annualized) for the current quarter. Following that quarter the real interest rate returns to steady state and remains there forever. The initial cut comes as a surprise, but subsequently the path of the real interest rate is known with certainty.

The left panel of figure 5 plots the impulse response function of output in the full model (solid blue line), the contribution coming from durable expenditures (dotted red line), and the output response in a standard new Keynesian model. In the full model, output expands on impact by 0.6%. But once the stimulus is removed at quarter $t = 1$, output falls below steady state by -0.2%. Subsequently, output gradually converges to steady state.

The red dotted line shows that these dynamics are largely a reflection of durable expenditure. Intuitively, a reduction in the real interest rate reduces the opportunity cost of owning a durable. This induces households near the durable adjustment threshold to accelerate their durable purchases, which results in an increase in aggregate demand and output at $t = 0$. However, households that adjust at $t = 0$ are no longer interested in adjusting at $t = 1$ so durable demand falls below its steady state level at $t = 1$ and in subsequent quarters.

The intertemporal shifting of aggregate demand is thus an intuitive outcome of our full model that stands in contrast to the standard three-equation new Keynesian model, in which nondurable expenditure is the only component of aggregate demand. The dashed blue line in Figure 5 shows that the response of output in that model displays no reversal once we remove monetary accom-
Figure 5: The left panel shows the percentage change in output following a 1% reduction in the real interest rate at time $t=0$ in the full model with durables (solid line) compared to the standard new Keynesian model (dashed line). The right panel shows the corresponding cumulative change in output, i.e., the integral of the impulse response function in the left panel.

In the right panel of Figure 5, we plot the corresponding cumulative increases in output, which are the integrals under the impulse response functions in the left panel. The solid blue line for the full model shows that almost two thirds of the increase in output at $t = 0$ is subsequently reversed. The reversal is almost entirely accounted for by durable expenditure. In the long run, the distribution of durable holdings will return to the steady state distribution so there is a strong tendency for the purchases that are accelerated into date $t = 0$ to be offset by missing purchases going forward.

This example featured a sharp change in intertemporal prices between quarters 0 and 1, which makes intertemporal shifting of aggregate demand obvious to see. In this sense, the example has more in common with the episodes in Section 3.3 (such as Cash-for-Clunkers), than the monetary shocks in Section 3, which we estimate to be very persistent so accommodation is only gradually reduced. In our model, the gradual reduction in accommodation obscures the reversal.

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22 We simulate the three-equation model assuming an intertemporal elasticity of substitution equal to $\sigma^{-1} = 2.46$, which implies that the interest rate cut at $t = 0$ has the same output effect on impact as in the full model.

23 The main reason that the increase in durable purchases is not fully reversed is that the monetary policy shock increases the number of adjustments that occur and each adjustment incurs adjustment costs that are counted as part of durable expenditure. The cumulative number of adjustments is not fully reversed because some households will be prompted to adjust by monetary policy and then soon after experience shocks that would induce another adjustment regardless of the monetary accommodation. In those situations monetary policy has created an extra adjustment.
5.2 The Power of Forward Guidance

In choosing to accelerate an adjustment, a household is giving up the interest on savings that it would have earned until the adjustment would have otherwise occurred. Therefore, the opportunity cost of accelerating a durable purchase includes the interest foregone over the period between the new purchase date and the original purchase date. The households that are induced to adjust their durables by monetary policy are those that were already near the adjustment threshold and therefore likely to adjust soon even in the absence of monetary stimulus. Therefore, the opportunity cost of adjusting the timing is primarily affected by short-term interest rates. By contrast, forward interest rates at horizons beyond the time the household was already planning to adjust do not materially change the incentives to accelerate the timing of a purchase.

By this logic, the extensive margin of durable demand is particularly sensitive to short-term interest rates and less sensitive to forward interest rates. As a result, lumpy durables are an effective way to mitigate the “forward guidance puzzle” that arises in standard New Keynesian models. These models predict promises of lower interest rates arbitrarily far in the future have the same output effect today as contemporaneous interest rate cuts (Del Negro, Giannoni, and Patterson, 2015; McKay, Nakamura, and Steinsson, 2016).

To demonstrate that current interest rates are more powerful than future interest rates in stimulating current durable demand, we simulate two different paths for the real interest rate. Each path cuts the interest rate by 1 percentage point (annualized) for a single quarter, but the paths differ in the timing of the cut. The first path cuts the current rate between quarters 0 and 1, the second cuts the rate between quarters 8 and 9.

Figure 6 shows the results. Cutting the contemporaneous interest rate by 1% at \( t = 0 \) is about three times more effective at stimulating demand at \( t = 0 \) than a 1% cut to years from now. By contrast, the standard three equation model predicts that current and future interest changes have the same effect on demand at \( t = 0 \)—they are perfectly substitutable in this regard. This is shown by the opaque, dashed lines in Figure 6. The cumulative output effects of forward guidance (the integral under the impulse response) are also significantly smaller in our model than in the standard three-equation model. A promise of a 1% cut at \( t = 8 \) yields a cumulative output increase between \( t = 0 \) and \( t = 8 \) of 2.0% in our model, about one third of the 5.5% increase in the three-equation

\[ 24 \text{For a household that needs to borrow to finance the durable, the opportunity cost includes the additional interest paid by pulling the purchase forward.} \]
Figure 6: Output responses to real interest rate cuts of 1% (annualized) that last for one quarter at horizons of 0 and 8 quarters. Full model refers to our model with lumpy durables. Three equation model refers to the standard New Keynesian model, calibrated to yield the same impact effect on output for the contemporaneous interest rate cut. The simulation includes endogenous changes in aggregate income and the relative durable price. Perfect foresight transition paths.

Durable goods demand is the key source of difference between our model and the three equation model. If we modify our model to have only non-durable consumption and a larger intertemporal elasticity of substitution, the output response to forward guidance looks very similar to the three equation model because the model is then close to the incomplete-markets irrelevance case in Werning (2015).²⁵

To be clear, forward guidance still has important effects on output in our model; they are simply much weaker than those implied by the standard three equation model.

6  \( r^* \) in the Great Recession

We now turn our attention to evolution of \( r^* \) during and after the Great Recession. As forward guidance is less effective in our model than in the standard New Keynesian model, interest rates needed to move by more to offset the shocks hitting the economy implying \( r^* \) fell by more. Due to the intertemporal shifting effects of monetary policy, the Federal Reserve’s stimulus during the Great Recession led to a lower \( r^* \) than would have been observed without forward guidance.²⁵

²⁵See Appendix Figure A.6.
Great Recession lead to persistent weakness in demand that kept $r^*$ low for many years to come.

### 6.1 Definition of $r^*$

We define $r^*$ as the real interest rate that is consistent with a zero output gap. To implement this definition we must account for the fact that the current output gap depends not just on the contemporaneous interest rate but also on expectations of future real interest rates. Therefore, at date $t$ we seek a path for real interest rates going forward that is consistent with a zero output gap at $t$ and an expectation that the output gap will remain zero going forward.

The path of $r^*$ depends on the state of the economy and evolves over time as the state evolves. The distribution of durable holdings is an important part of the state. For instance, suppose households have too many durables relative to their desired stocks. There will then be a short-fall of durable demand at steady state interest rates and $r^*$ will fall. The logic of intertemporal shifting is that the distribution of durable holdings evolves in response to monetary policy actions, which then leads to changes in $r^*$ going forward. For instance, monetary accommodation today leads households to enter the next period with more durables than they otherwise would have.

In order to demonstrate how alternative courses of monetary policy lead to different paths for $r^*$ we introduce three cases. In the first, we assume that the central bank fully accommodates shocks to maintain a zero output gap at all times. This case corresponds to the evolution of the real interest rate in a flexible-price economy and is often considered in the literature (e.g., Smets and Wouters, 2007). In the second, we assume that the central bank provides the accommodation observed in the data. In the Great Recession the Federal Reserve did not provide as much accommodation as would have been needed to maintain a zero output gap. As a result, fewer durable purchases were accelerated and going forward $r^*$ is higher than the full-accommodation case. In the third case we suppose the Federal Reserve does not respond to shocks at all during the years 2007 through 2012. In this case, monetary accommodation does not accelerate any durable purchases and this shapes the distribution of durable holdings going forward. Appendix G formally defines our counterfactuals.

To compute $r^*$ we use an algorithm that builds on recent work by Auclert et al. (2019). The main step in the algorithm is to compute the impulse responses of $r^*$ following the aggregate shocks. Having done that, we can simulate $r^*$ in response to a sequence of shocks by accumulating
the relevant impulse responses. Here, we describe the algorithm to compute the response of $r^*$ to a
shock through an example and provide a more thorough explanation in Appendix G. Suppose there
is a transitory decrease in government spending by 1% of steady state output at quarter $t = 0$. Let
the vector $Y_G$ be the impulse response of the output gap under constant real rates. In this example,
$Y_G = [1, 0, 0, ..., 0]'$. Let $A_Y$ be a matrix where the $(i, j)$ element gives the response of the output
gap at horizon $i − 1$ to a change in the real rate at horizon $j − 1$. The path for real interest rates
that closes the output gap along the transition path is then $r^* = \bar{r} − A_{Y}^{-1}Y_G$.

Figure 7 shows this full-accommodation $r^*$ path following the transitory shock to government
spending. Closing that output gap requires the central bank to reduce interest rates at $t = 0$, which
leads households to accelerate durable spending and increase aggregate demand. Since future
durable demand shifts towards the present, keeping the output gap at zero requires subsequent
reductions in real interest rates.

In Figure 7 we also plot the $r^*$ path if the central bank provided no accommodation at $t = 0$ (red
dashed line). In this case no demand is shifted intertemporally, the output gap is zero for $t \geq 1$,
and no monetary action is required going forward.\textsuperscript{26} By comparing the no-accommodation scenario
with the full-accommodation scenario, it becomes evident that the accommodative monetary policy
at $t = 0$ causes the decline in $r^*$ going forward.

6.2 The Great Recession Through the Lens of Our Model

To calculate $r^*$, we must extract the shocks that account for the aggregate time series during the
Great Recession. We seek to match four aggregate time series from 1991-2019:\textsuperscript{27} the output gap, $\hat{Y}_t$,
constructed using the CBO’s estimate of potential output; the change in the durable expenditure
share (relative to potential GDP) $s_t^x = \frac{p_t x_t}{Y_{pot}^t}$; the demeaned real interest rate, $r_t$; and the demeaned
spread of the 30-year mortgage rate over the ten-year treasury yield, $r_t^b$.\textsuperscript{28} We will extract four
shocks from these series: the permanent productivity shock $\epsilon^Z_t$ (equivalent to a permanent income

\textsuperscript{26}In this case, the reduction in income at $t = 0$ is perfectly offset by a reduction in taxes so the households
experience no change in income or interest rates in period 0 and it is as if the economy had remained in steady state.

\textsuperscript{27}We chose 1991 as a starting date for three reasons. First, it is sufficiently distant from the Great Recession that
the initial state of the economy should have little effect on the dynamics of the economy during the Great Recession.
Second, the real rate displays no trend from 1991 through 2007, which side-steps issues for how to detrend the real
rate. Third, the persistence of inflation is roughly constant after 1991, and this is a key input the determination of
non-durable inflation expectations and thus the ex ante real rate. (The persistence is much higher before 1991.)

\textsuperscript{28}We demean the series using the mean from 1991-2007, so as to not incorporate the downward-trend in the real
rate during the Great Recession.
Figure 7: $r^*$ given an unanticipated and transitory decrease in government spending of 1% of steady state output at time $t = 0$. The solid blue line plots the full accommodation $r^*$, whereas the dashed red line plots $r^*$ when monetary policy does not accommodate at $t = 0$.

shock), the non-household demand shock $\epsilon^G_t$, the monetary policy shock $\epsilon^r_t$, and the shock to the borrowing spread $\epsilon^{rb}_t$.

We use a novel filtering approach that we describe in detail in Appendix F. For each of the shocks we construct the impulse response functions of $\{\hat{Y}, s^x, r, r^{rb}\}$, which are reported in Appendix H.

We then proceed recursively: at each date $t$, we create a forecast for $\{\hat{Y}_t, s^x_t, r_t, r^{rb}_t\}$ based on all the previous shocks we have filtered. We then solve for the shocks at date $t$ that explain the difference between the data observed at date $t$ and our forecast. We again make use of the assumption that the economy’s dynamics are linear in the shocks to perform this calculation. Specifically, the forecast for the data is a convolution of the previous shocks and the impulse response functions and solving for the date $t$ shocks requires inverting a matrix of the impact response of each data series with respect to each shock. We initialize this procedure in 1991 assuming the economy is in steady state. This filtering method is equivalent to the Kalman filter (and also the Kalman smoother) given that

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29In this section, we label the government spending shock “non-household demand shock” since its role is to account for the residual output gap that cannot be explained by the shocks to households.

30We directly estimate the persistence of the borrowing spread over 1991-2007, which yields $\rho_{rb} = -0.63$ and is equivalent to a quarterly persistence of 0.85. We set the persistence of the non-household demand shock $\rho_g = -0.9$ equivalent to a quarterly persistence of 0.8. We deliberately choose a value at the lower end of the persistence spectrum typically estimated for demand shocks, since a greater persistence only increases the importance of intertemporal shifting. For example with a quarterly persistence of 0.9, intertemporal shifting contributes 3.2 percentage points to the decline in the 2-year natural rate of interest, as opposed to 2.8 percentage points in our baseline calibration.
there is no measurement error and the initial state is known with certainty (see Appendix F). The benefit of this approach to filtering is that it relies only on impulse response functions and does not require a state transition matrix, which is not readily available for a heterogeneous agent model in which the state includes a distribution.

The shocks are uniquely identified as they imply very different impulse response functions for $\{\hat{Y}, s^x, r, r^b\}$. A permanent decline in productivity causes a durable overhang, with a large reduction in durable spending, a negative output gap, and a reduction in the real rate as the central bank accommodates. A negative non-household demand shock causes a negative output gap along with an increase in durable spending to potential GDP as accommodative monetary policy stimulates durable expenditure. A contractionary monetary policy shock causes a negative output gap and reduces durable spending relative to potential GDP, accompanied by an increase in the real rate. Finally, a shock to the borrowing spread is easily identified as it is the sole source of variation in $r^b$.

We plot our filtered shocks in Figure 8. Our procedure identifies large negative productivity shocks during the Great Recession owing to the weakness in durable spending. A permanent decline in productivity is akin to an overbuilding shock in the sense that the economy now has more durables than it would like. With some delay there is also a fall in non-household demand as the productivity shocks are not sufficient to explain the decline in the output gap. Borrowing spread shocks are prevalent in the run-up to the financial crisis, but the level of spreads has largely normalized by the end of the Great Recession. Monetary policy shocks tend to be positive throughout the Great Recession, as the decline in real interest rates is smaller than the model expects given the other shocks. This reflects the limited accommodation the Federal Reserve was able to provide given the zero lower bound constraint.

6.3 $r^*$ in the Great Recession

In Figure 9 we plot the time series for 2-year natural rate of interest implied by our model. As we mentioned above, our $r^*$ calculation generates an entire path for real interest rates at each date. We summarize this path using the 2-year horizon because it captures the stance of policy, including forward guidance, better than the short rate, while still being centered on the horizon
Figure 8: The shocks are filtered such that the model exactly replicates the aggregate time series of the output gap, the real rate of interest, and the change in durable spending as a fraction of potential output.

Over which monetary policy is typically conducted.\textsuperscript{31} We stop incorporating new shocks in 2013, so that the paths for $r^*$ beginning in 2013 are forecasts based on the shocks up to 2012Q4. We look at forecasts during the recovery to argue that the events that took place during the Great Recession can explain the very gradual normalization of interest rates in the recovery. Forecasts make clear that the low rates after 2012 are due to the after-effects of the recession and not due to new shocks. We begin forecasting in 2013 because we interpret 2012Q4 as the end of the period in which there was the most monetary accommodation. The 5-year TIPS yield reached a minimum in 2012Q4 and then rose substantially early in 2013 as the Federal Reserve indicated it would soon scale back its large-scale asset purchases.

Under the full-accommodation scenario, the 2-year $r^*$ drops from 1.9% above steady state in 2006Q4 to -5.8% below steady state in 2011Q2. From there, $r^*$ only slowly recovers to steady state, even though we do not incorporate new shocks after 2012. In 2013Q1 the 2-year natural rate is $-5\%$ below steady state, in 2015Q1 it is $-3.5\%$. Only in 2019Q2, more than ten years after the onset of the Great Recession, does the 2-year natural rate rise to within 1% of the steady state. Thus, the

\textsuperscript{31} All our model-based long-term interest rates are computed according to the expectations hypothesis. Results for 5-year and 10-year maturities show similar patterns to what we show here for the 2-year yield. The model predicts that the accommodation will be provided mostly on the shorter end of the yield curve and so movements in longer-term yields are attenuated relative to the 2-year yield.
cyclical factors during the Great Recession imply a substantial and very persistent decline in the natural rate of interest.

The persistent effects on $r^*$ come from two main forces. First, the permanent productivity decline causes a durable overhang as households would want to hold fewer durables given their lower income. This causes a persistent drop in durable expenditures, which requires persistent monetary accommodation. Second, accommodating these shocks requires strong monetary accommodation for an extended period of time. Since low interest rates pull forward aggregate demand, the central bank must keep interest rates low for an extended period to avoid this intertemporal shifting to cause negative output gaps in the future.

To separate the importance of the fundamental shocks from the importance of intertemporal shifting in the dynamics of $r^*$, we plot a scenario in which the central bank does not accommodate the shocks from 2007 up to and including 2012 (dashed black line) and then resumes full accommodation in 2013. Thus, both the full-accommodation and this “no-accommodation” scenarios feature zero output gaps going forward in 2013, but these scenarios condition on different histories of policy. In the full-accommodation scenario the central bank pulls demand forward from 2013 and beyond to close output gaps from 2007-2012, whereas this does not occur in the no-accommodation scenario.\footnote{Under “no accommodation” monetary policy proceeds on the course that was expected at the end of 2006 until the start of 2013.}
As a consequence, the natural real rate of interest falls from 1.9% above steady state in 2006Q4 to -0.6% in 2013Q1 in the no-accommodation scenario. Since the full-accommodation $r^*$ dropped to $-5\%$, this implies that 4.4 percentage points of the decline in $r^*$ under full accommodation is due to intertemporal shifting.

In practice, the central bank provided less than full accommodation during the recession so $r^*$ was not as low in 2013 as implied by the full-accommodation scenario. The extent to which $r^*$ declined under the actual accommodation provided is instead captured by the red dashed line. In this case, the 2-year natural rate drops from 2.1% above steady state in 2006Q4 to $-3.6\%$ in 2013Q1. Comparing to the no-accommodation scenario, 2.5 percentage points of the drop can be explained as coming directly from the shocks, with the remaining 3.2 percentage points due to intertemporal shifting from accommodative monetary policy. There remains a substantial difference between the no-accommodation and actual-accommodation scenarios for several years to come and there is still a noticeable difference at the end of the sample.

These results show the accommodative monetary policy provided by the Federal Reserve during the Great Recession had a large and persistent effect on $r^*$. This finding stands in contrast to the prevailing neo-Wicksellian view that $r^*$ is largely exogenous to monetary policy, and that policy must track this $r^*$ path.

### 6.4 The Gradual Normalization of Interest Rates

We have showed that $r^*$ was considerably lower during the recovery as a result of the monetary stimulus provided in the Great Recession. We now show that the forecasted evolution of $r^*$ in the recovery lines up with the evolution of real interest rates that materialized. As we will describe, the weaker power of forward guidance and the intertemporal shifting are important in causing $r^*$ to fall as much as it did, as well as allowing the model to explain the gradual normalization of interest rates in the data.

In the left panel of Figure 10 we compare the contemporaneous $r^*$ in the model with the contemporaneous real rate in the data, and in the right panel we compare the 5-year $r^*$ from the model with the 5-year TIPS yield.\(^{33}\) In both graphs we stop incorporating shocks in 2012Q4, so

\(^{33}\) We use the 5-year TIPS for this comparison, because this is the shortest maturity issued by the Treasury and shorter maturities are notoriously illiquid and therefore the yield data is less reliable at shorter maturities. We normalize the TIPS yield to equal our model $r^*$ in 2005Q1, when our model forecasts the smallest absolute output gaps over the next 10 years.
Figure 10: Left panel: short-term (contemporaneous) $r^*$ under actual accommodation from the model (forecasted after 2012Q4), and ex-ante real interest rate in the data. Right panel: The 5-year $r^*$ under actual accommodation (forecasted after 2012Q4), as well as the 5-year TIPS yield. The latter is normalized to equal $r^*$ under actual accommodation in 2005Q1, when our model forecasts the smallest absolute output gaps over the next 10 years.

the model-based natural rate of interest for 2013Q1 onward is a forecast as of 2012Q4.

$r^*$ is predicted to recover only very slowly from its low level in 2012Q4, both at short and long ends of the yield curve. Thus the low level of rates observed in 2013 and for many years to come can be explained by past shocks and the associated monetary accommodation. For example, the contemporaneous $r^*$ is -3.3% below steady state in 2015, -2.3% in 2017, and -1.2% in 2019.

The recovery of the 5-year $r^*$ is similarly slow, and it aligns well with the normalization of long-term TIPS yields. Thus the model is able to accurately forecast the evolution rates at multiple points along the yield curve.

That the model predicts a low level of $r^*$ up to 2013 is perhaps less surprising. We filter the data so that the model exactly matches the contemporaneous real rate in the data, and since there is a negative output gap, the model predicts that real rates (both at the short and long end of the yield curve) ought to be even lower to close the output gap.

Overall, our model suggests that cyclical factors occurring before 2013 are important in explaining the persistently low levels of interest rates during and after the Great Recession and the late lift-off of interest rates in December 2015. Our work suggests that one does not need to appeal to secular forces, such as demographics, to explain the behavior of real interest rates over this period.

We next show that both intertemporal shifting and the weaker power of forward guidance
are important for our inference that $r^*$ is expected to be persistently low after 2012. Intuitively, intertemporal shifting implies that the real rate must be kept low for a prolonged period of time. Similarly, the weaker power of forward guidance implies that expected rate path must be lower to provide the same amount of stimulus.

We demonstrate this in Figure 11. The blue solid line is the full-accommodation 5-year $r^*$ in our baseline model, which shows a very gradual normalization after 2012. We then consider two alternative scenarios for how the monetary transmission mechanism works. In the first, we conduct our analysis (including filtering the shocks), using the three-equation New Keynesian model (cyan dotted line). This model differs both in that it makes forward guidance more powerful and in that there are no intertemporal shifting effects. To separate the roles of these two differences, our second scenario assumes that the effects of forward guidance are those predicted by our full model, but there are no intertemporal shifting effects (red dashed line).34

In these alternative scenarios monetary policy is more powerful, in that the central bank is able to stabilize the economy in the aftermath of the Great Recession without keeping interest as low for as long. In both scenarios, $r^*$ falls by less and normalizes more quickly. Forward guidance effects are slightly more important earlier in the recession, whereas intertemporal shifting accounts for a bit more of the persistence. But both features are quantitatively important to explain the persistently low natural rate of interest after the Great Recession.

These results show that there is nothing inherent in our analysis that necessarily implies a large fall and slow normalization of $r^*$ after the Great Recession. Rather, it is because the model implies that monetary policy is relatively weak that $r^*$ falls so far and because the model has strong internal propagation of the reduction in aggregate demand that $r^*$ remains persistently low.

6.5 Robustness

We first examine a case were monetary policy is only half as effective at closing the output gap as we assume in our benchmark calculation. This is meant to capture the state dependence described by Berger and Vavra (2015) in which durable demand is less sensitive to stimulus in a recession. We have explored the non-linearity in our model and found that the response of output to monetary

34 Formally, we manipulate the monetary news matrix in which the $(i, j)$ element is the effect of lower real interest rates at a horizon $j - 1$ on the output gap at horizon $i - 1$. We eliminate reversal effects by setting all elements below the diagonal to zero, so past monetary policy has no effect on future output.
Figure 11: Full accommodation 5-year $r^*$ in our baseline model (blue solid line) against assuming monetary policy is not subject to reversals (red dashed line) and monetary policy also features forward guidance as powerful as in a standard new Keynesian model (yellow dash-dotted line). In each case we filter the data given the assumptions about monetary policy, so each model matches exactly the time series for the output gap, the contemporaneous real interest rate, the change in durable expenditure share to potential GDP, and the borrowing spread. From 2013Q1 onward the reported 5-year $r^*$ is a forecast based on the history of shocks up to 2012Q4.

The stimulus is about 40% smaller in the depth of a large recession similar in magnitude to the Great Recession. Less powerful monetary policy has two implications for our analysis: interest rates need to move more to close an output gap of a given size, but the shocks we filter out of the data are smaller.35 These effects cancel to some extent, but the first effect is slightly stronger than the second implying a larger drop in the natural rate of interest as we show in the left panel of Figure 12. Because these larger shocks require more monetary accommodation, the contribution of intertemporal shifting to the persistent decline in $r^*$ increases to 3.8 percentage points.

The second robustness analysis we conduct uses a different specification for how agents expect the real interest rate to evolve. Our analysis does not explicitly incorporate an ELB constraint. Rather, in our filtering we match the behavior of the real interest rate during the Great Recession using monetary policy shocks. The limit the ELB placed on the movement of the real rate is replaced with contractionary monetary shocks and agents may expect that the ELB will be violated as the

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35To see this logic, suppose the output gap is given by $\hat{Y} = -ar + \epsilon + \nu$, where $a$ is a parameter that controls the strength of monetary policy and $\epsilon$ and $\nu$ are two shocks. Suppose that the central bank fully responds to $\epsilon$ but does not respond to $\nu$ so $r = \epsilon/a$. To close the output gap we need $r^* = (\epsilon + \nu)/a$. We observe $r = \epsilon/a$ and the output gap $\hat{Y} = \nu$ and plugging these in yields $r^* = r + \hat{Y}/a$. So the parameter $a$ affects the part of $r^*$ that is associated with the output gap that is not closed in practice, but not the movements in $r^*$ that the central bank responds to.
Figure 12: The left panel shows the time series of the 2-year $r^*$ under full accommodation, actual accommodation, and no accommodation from 2007-2012, when monetary policy is only half as effective at stimulating aggregate demand than in our baseline model. The right panel shows the corresponding outcome when monetary policy does not endogenously respond to shocks and we match the time series of the real interest rate with monetary policy shocks alone. In each case we stop incorporating new shocks after 2012Q4, so the figures plot the expected path of the natural rate from 2013Q1 onward.

monetary shocks dissipate. As an alternative we consider a specification in which the model’s real rate is entirely exogenous and follows an AR(1).\footnote{The estimated quarterly persistence is 0.966.} In this case, agents always expect the real rate will revert towards its steady state level. In the Great Recession this means agents expected real rates to gradually rise moving the economy away from the level implied by the ELB. Note that this change only affects our filtering step—specifically, it affects the path for the output gap in the absence of monetary accommodation—and does not affect our $r^*$ calculation for a given expected output gap path. The right panel of figure 12 shows the results. The drop in $r^*$ is smaller than in our baseline, but the overall contribution of intertemporal shifting to the decline in $r^*$ is slightly smaller at 2.9 percentage points as opposed to 3.2 p.p. in the baseline model. These results reflect the fact that monetary policy provides slightly less accommodation through the path of expected real rates.
7 Conclusion

We investigate the implications of the micro-foundations of durable goods demand for monetary policy. In our heterogeneous-agent model with fixed costs on durable adjustments, a key monetary transmission mechanism is to accelerate the timing of durable goods purchases. A first consequence of this channel is that the central bank faces an intertemporal trade-off for aggregate demand, as current stimulus reduces future demand. Second, the stimulative power of forward interest rates is reduced, as households make a short-term decision of whether to adjust now or in the near future.

These aspects of the monetary transmission mechanism have important implications for the evolution of $r^*$ during and after the Great Recession. The reduced effectiveness of monetary policy due to the weaker power of forward guidance implies that the natural rate of interest measured as a 2-year or 5-year yield falls by more. Simply put, policymakers need to lean more heavily on the levers of policy if those levers are less effective. In the aftermath of the Great Recession, real interest rates remained very low for years to come and our analysis attributes much of these interest rate dynamics to the intertemporal shifting effects of monetary policy.

The view we put forward here, in which $r^*$ responds quite strongly to changes in monetary policy, contrasts with the neo-Wicksellian paradigm, in which $r^*$ is generally considered to be independent of monetary policy. One implication of our analysis is that monetary accommodation has a side effect of reducing future policy space by reducing $r^*$ towards the lower bound implied by the ELB. While our analysis focuses on a positive description of the economy, the intertemporal shifting of demand we highlight may have important implications for optimal monetary policy.
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A Computational Appendix

We solve the model building on the routines available from Benjamin Moll’s website http://www.princeton.edu/\%7Emoll/HACTproject.htm and described in Achdou et al. (2017).

A.1 Steady state

Define $k = a + \lambda d$ as the distance from the borrowing limit. Construct tensor grids over the state variables $(k, d, z)$. Then the steady state policy function is constructed as follows:

1. Start with an initial guess of the value function $v(k, d, z)$ and the value conditional on making an adjustment $v^*(k, d, z)$.

2. Solve for the optimal consumption and saving decisions when not adjusting. Compute $v_k$ both as a forward difference $v^f_k$ and as a backward difference $v^b_k$. At the boundaries of $v^f_k$ and $v^b_k$ impose that the drift of $k$ is zero. Invert $v_k(k, d, z) = U_c(c, d)$ to solve for $c^f(k, d, z)$ and $c^b(k, d, z)$, and the corresponding drift of $k$, $s^f(k, d, z)$ and $s^b(k, d, z)$. Finally, let $c^0(k, d, z)$ be the consumption consistent with zero drift. Pick among the candidates based on the following rule:

   (a) If $s^f < 0$ and $s^b < 0$ pick $c^b, s^b$.
   (b) If $s^f > 0$ and $s^b > 0$ pick $c^f, s^f$.
   (c) If $s^f < 0$ and $s^b > 0$ pick $c^0, s^0$.
   (d) If $s^f > 0$ and $s^b < 0$ pick the candidate that yields a larger value for the Hamiltonian.

Using the solution, compute the felicity function $u(c, d)$.

3. Construct the transition matrix $A$ based on the endogenous drifts of $k$ and the exogenous drifts and shocks to $d, z$. See Achdou et al. (2017) for details.

4. The HJB equation can now be written as $\min \{\rho v - u - Av, v - v^*\} = 0$, and solved using an LCP solver for $v$. We use Yuval Tassa’s solver http://www.mathworks.com/matlabcentral/fileexchange/20952.

5. Compute optimal choice of $d'$ conditional on adjusting and the corresponding $v^* = \max_{d'} v(k', d', z)$, where $k' = \lambda d' + k - (\lambda + f)d$. 

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6. Repeat steps 1-5 until convergence.

7. To obtain the steady state distribution, convert the policy functions for \( k' \) and \( d' \) conditional on an adjustment to index form. Fractions of an index determine the weights we assign to each index.

8. Create a matrix \( C^{\text{noadj}} = A - \text{diag}(\theta) \). Then set all the columns in \( C^{\text{noadj}} \) that correspond to adjustment points to zero. Define \( A^{\text{adj}} = A - C^{\text{noadj}} \). This matrix contains the mass at adjustment points that needs to be reallocated to the nodes of the optimal \( k', d' \). We assign this mass to the nodes surrounding \( k', d' \) based on the index fractions in the previous step. This yields a matrix of adjustments \( C^{\text{adj}} \). The transition matrix is then \( C = C^{\text{noadj}} + C^{\text{adj}} \).

9. Solve \( 0 = Cg \) for the steady state distribution \( g \).

A.2 Impulse response functions

1. Repeat steps 1-5, iterating backward from the terminal condition \( v_T = v \). Each iteration reduces \( t \) by \( dt \). Continue until \( t = 0 \) is reached.

2. Repeat steps 7-8 for each period of the IRF. The initial distribution \( g_0 \) requires a modification if \( p_0 \neq 1 \). The distribution of \( k \) needs to shift since \( k = a - \lambda pd \) and \( a, d \) are fixed in that instant.

3. For each \( t \), if the adjustment thresholds change, then all the mass in \( g_t \) that is in the new adjustment region must be immediately shifted to its new location using the procedure in step 8. Call the new distribution \( \tilde{g}_t \). Then compute \( g_{t+dt} = \tilde{g}_t + C_t \tilde{g}_t dt \). Repeat this step until \( t = T \).

4. In general equilibrium, update the durable price and income, and repeat the previous steps 1-3 until convergence. For the case where the real rate is endogenous, compute the Jacobian with respect to the real rate of durables \( r^d_t \) at each iteration step, and use the Newton method to update the real rate \( r^d_t \). From \( r^d_t = r_t \frac{p_t + dt}{p_t} \) calculate \( \{r_t, p_t, Y_t\}_{t=0}^T \) using (12), (10), (11), the terminal conditions \( r_T = \bar{r}, p_T = 1, Y_T = 1 \), and using \( Y_t = X_t + Y^{\text{smooth}}_t \), where \( Y^{\text{smooth}} = c + G + (r^b - r) \int_0^1 a_idI_{a_id < 0} \) is the solution for the (smooth) non-durable components of aggregate demand from the previous iteration.
B Estimation

The estimation strategy largely follows Berger and Vavra (2015). The details of the data selection and estimation algorithm below are meant to facilitate replication of our results.

B.1 Data

We use PSID data from 1999 through 2009.

B.1.1 Variables

Real non-durable consumption is nominal non-durable consumption in the PSID deflated by the BEA price index for non-durables (NIPA table 1.1.4). Nominal non-durable consumption is the sum of food expenditures, utility expenditures, home insurance, transportation expenditures, property taxes, health expenditures, child care expenditures, and education expenditures. We exclude any loan or lease payments from transportation expenditures to align the definition of non-durables with our model.

Real durable holdings are the sum of real house values and real vehicle values. Real house values are reported nominal house values deflated by the OFHEO national house price index. For renters we convert rent to a house value using the national house-to-rent ratio from Davis et al. (2008) available at http://www.aei.org/housing/land-price-indicators/. The PSID records the net wealth of up to three vehicles per household. We sum these values, add total vehicle debt (detailed below), and deflate the sum with the BEA price index for motor vehicles (NIPA table 1.2.4).

Real liquid asset holdings are the sum of cash and deposit holdings, stock holdings, and bond holdings, deflated by the non-durables price index.

We construct net real liquid assets by subtracting real debt from housing and vehicles. Mortgage debt is directly reported and we deflate it using the non-durables price index. We construct existing vehicle debt from the initial loan amount on all three cars and subtract the number of payments made times the average payment amount. In less than 1% of cases this results in a negative debt value, in which case we set vehicle debt to zero.

Housing adjustments come from either moving or a significant addition or repair. The PSID records the month and year of the most recent move since either the last interview (pre-2003) or
since January two years ago. If a move is recorded and the move falls after the previous interview, then we code it as a housing adjustment for the current wave; otherwise it is an adjustment in the previous wave. When the move falls in the month of the interview we break the tie based on whether the interview was in the first or second half of the month. For significant additions and repairs we record them as housing adjustments in the wave that they are reported.

Car adjustments are set to one if any one of the three reported cars has been acquired since the previous wave. This is the case if the most recent car’s acquisition date is after the previous wave’s interview date, or (if there is insufficient information using the date) a new car has been acquired less than three years ago and it was not reported in the previous wave.

The sample weight is the household weight in the PSID.

B.1.2 Sample selection

We only keep head of households since the data is reported at the household level. We drop heads of households 21 and younger, as well as households present for fewer than 3 waves. This selection helps with the estimation of household fixed effects. We drop households with zero durable holdings, or those with missing information on any variable. We winsorize all variables at the 5th and 95th percentile.

B.1.3 Household fixed effects

We demean durable holdings by the households average durable holdings over the sample. This accounts for permanent differences in tastes for durables across households, which are not part of the model. We also divide non-durable consumption, liquid asset holdings, and real debt holdings by a household’s average non-durable consumption over the sample. This helps account for permanent differences in income, which are again not part of the model.

B.1.4 Consistency with national aggregates

We divide all variables by average non-durable consumption in the sample. We then multiply each scaled variable (durables, liquid assets, debt) by a factor so that the sample average aligns with national aggregates from the fixed asset tables (durable-to-non-durable-consumption ratio) and the flow of funds (liquid-asset-to-non-durable-consumption and debt-to-non-durable-consumption).
The rescaling is necessary because the PSID collects data for 72% of non-durable expenditures on average (Li et al., 2010). Further, households appear to overestimate the value of their vehicles (Czajka et al., 2003).

### B.2 Estimation Algorithm

The steps of the algorithm are as follows:

1. Solve the model for a given intensity of match quality shocks $\theta$.

2. Forecast the probability of adjustment $P(a, d, y)$ for each initial state $(a, d, y)$ over the next two years. Also forecast the average non-durable consumption expenditure (including operating and maintenance costs) $\bar{c}$ for each initial state $(a, d, y)$ over the next two years. From the latter we obtain a steady-state distribution $G(a, d, \bar{c})$.

3. Regress the optimal durable stock $d^*$ in the model on $a, a^2, d, \bar{c}, d/\bar{c}$ weighted using the steady-state distribution. The vector of estimated coefficient is $\beta$.

4. Add measurement to the model variables $a, d, \bar{c}$ using three independent Gaussian quadratures, one for each variable. This yields a new distribution $\hat{G}(a, d, \bar{c})$ which includes measurement error. Note that the adjustment probabilities are based on the true underlying $G(a, d, \bar{c})$.

5. Compute gaps $\omega = d^* - d$ for each point in the distribution $\hat{G}$. Integrating over $\omega$ using $\hat{G}$ yields the pdf $f(\omega)$ in the model. Similarly integrating the probability of adjustment $P(a, d, y)$ over $\omega$ using $\hat{G}$ yields the hazard rate $h(\omega)$ in the model.

6. In the data combine reported $a, d, \bar{c}$ and our estimates $\beta$ to predict $d^*$ and the durable gap $\omega = d^* - d$. Use the sample weights to compute $f(\omega)$ and the adjustment hazard $h(\omega)$.

7. Compute loss function $L = \sum \omega \cdot w(\omega) \cdot ||f_{model}(\omega) - f_{data}(\omega)|| + ||h_{model}(\omega) - h_{data}(\omega)||$ where the weight is $w(\omega) = \frac{1}{4}(f_{model}(\omega) + f_{data}(\omega))(h_{model}(\omega) + h_{data}(\omega))$. This weighting function attaches more weight to bins the greater the fraction of adjustments accounted for by that bin. Conversely, we attach little weight to regions in which both model and data predict few adjustments.
8. Repeat steps 4, 5, and 7 using a range of values for the standard deviation of the measurement error. Then pick the value that results in the smallest loss in 7.

9. Repeat steps 1-8 using a range of values for $\theta$. Pick the $\theta$ with the smallest loss in 8.

10. To construct standard errors, sample 1000 new datasets with replacement from the original dataset. Repeat steps 6 and 7 for each dataset, record the loss-minimizing value for $\theta$ and the associated density and hazard function from both data and model.

C Data Appendix

C.1 Variables for Impulse Response Functions

In this section we detail how we construct the variables for the empirical impulse response functions to monetary policy shocks in figures 2 and 3 of section 3.

The base series the set of variables in Table 4, which we download from the St Louis FRED database.

To construct real durable and non-durable expenditure, we first construct their respective price indices. The problem is one where we have two components of nominal expenditure $y_t = x_{1t} + x_{2t}$ (e.g., durable expenditure equals consumer durables plus residential investment), and their respective price indices $p_{1t}$ and $p_{2t}$. We want to construct the price index $p_t$ for $y_t$. We first construct average annual nominal expenditure $y^a_t = \frac{1}{4} \sum_{t \in \text{year} = a} y_t$, and the shares of the components, $s^a_{1t} = \frac{x^a_{1t}}{y^a_t}$ and $s^a_{2t} = \frac{x^a_{2t}}{y^a_t}$. We regress one of the annual shares on an annual quadratic time trend, create the predicted values $\hat{s}^a_{1t}$, and use it as a weight, $p_t = \hat{s}^a_{1t}p_{1t} + (1 - \hat{s}^a_{1t})p_{2t}$. Real expenditure is then calculated as $y_t/p_t$. We convert all real expenditure to per capita by dividing by population.

We follow this procedure for both durables (consumer durables plus residential investment) and non-durables (consumer non-durables plus services net of housing).

For the price series of residential investment and consumer services we make specific modifications. We separate residential investment into investment into new structures and other residential investment. For investment into new structures we use the FHFA national house price index to capture changes in the price of land as well as the price of the new structure. For other residential investment we use that respective price index from the BEA. The weights are based on nominal expenditures in new residential structures and other residential investment and calculated as above.
<table>
<thead>
<tr>
<th>Variable Name</th>
<th>FRED Series Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>B230RC0Q173SBEA</td>
</tr>
<tr>
<td>Income (GDP)</td>
<td>GDPC1</td>
</tr>
<tr>
<td>Federal Funds Rate</td>
<td>FEDFUNDS</td>
</tr>
<tr>
<td>Consumer Durable Expenditure</td>
<td>PCDG</td>
</tr>
<tr>
<td>Residential Investment</td>
<td>PRFI</td>
</tr>
<tr>
<td>Consumer Non-durable Expenditure</td>
<td>PCEND</td>
</tr>
<tr>
<td>Consumer Service Expenditure</td>
<td>PCES</td>
</tr>
<tr>
<td>Consumer Housing Services Expenditure</td>
<td>DHSGRC0</td>
</tr>
<tr>
<td>Durable Price Index</td>
<td>DDURRD3Q086SBEA</td>
</tr>
<tr>
<td>Residential Investment Price Index</td>
<td>B011RG3Q086SBEA</td>
</tr>
<tr>
<td>Non-durable Price Index</td>
<td>DNDGRG3M086SBEA</td>
</tr>
<tr>
<td>Services Price Index</td>
<td>DSERRG3M086SBEA</td>
</tr>
<tr>
<td>Services Price Index: Housing</td>
<td>DHUTRG3Q086SBEA</td>
</tr>
<tr>
<td>Consumer Expenditure: Motor Vehicles</td>
<td>DMOTRC1Q027SBEA</td>
</tr>
<tr>
<td>Motor Vehicles Price Index</td>
<td>DMOTRG3Q086SBEA</td>
</tr>
<tr>
<td>House Price Index</td>
<td>USSTHPI</td>
</tr>
<tr>
<td>Residential Investment: Permanent Site</td>
<td>A943RC1Q027SBEA</td>
</tr>
<tr>
<td>Residential Investment: Other</td>
<td>A863RC1Q027SBEA</td>
</tr>
<tr>
<td>Residential Investment Price Index: Other</td>
<td>A863RG3Q086SBEA</td>
</tr>
</tbody>
</table>

Table 4: Variable names and FRED series code.

For consumption of services we remove housing services because housing services in the model are obtained from \( h_t \) and not counted in \( c_t \). To do so we follow the same procedure as above for the housing and non-housing component of services. But rather than adding, we subtract the housing component, \( y_t = x_{2t} - x_{1t} \), and the price index is calculated as \( p_t = (p_{2t} - \bar{s}_{1t}p_{1t})/(1 - \bar{s}_{1t}) \).

The relative price series for durables is the price of durables divided by the price of non-durables and services. The real interest rate is defined in terms of non-durables. It is the federal funds rate net of realized non-durable inflation from this quarter to the next quarter.
C.2 PSID: Housing Adjustment Probability

This section details how we construct the time series for the probability of housing adjustment using the Panel Survey of Income Dynamics (PSID). We use these data to measure the extensive margin response of housing to monetary shocks in table 2 of section 3.

C.2.1 Sample

We keep only people who are heads of household and those who are in the Survey Research Center (SRC) sample. We only use PSID data from 1969-1997 when the survey frequency is annual.

C.2.2 Adjustment Series

We use the moved since spring series to create a record of adjustments. If moved since spring is true, we record an adjustment for that year. If moved since spring is false, we record no adjustment for that year. If the value is missing, we stop counting until the next time an adjustment is recorded, then count from there.

**Tenure Status** Tenure status refers to knowing what type of housing (rent or own). We use the series from the PSID labeled own/rent or what. There are also values for “other” or “not applicable.” If the values are “other” or “not applicable” we mark the tenure status as missing.

To align with Bachmann and Cooper (2014), we set values to missing if the observation does not have a tenure status or is lag does not have a tenure status. For example, if their observation is in the year 1992, we will set the adjustment series to missing if we do not know whether the head of household was owning or renting in either 1991 or 1992.

C.2.3 Weights

To weight observations in the regression, we take the mean of the family weight provided associated with the head of household across all periods for which we have year to year housing adjustments (1969-1997).

C.2.4 Time Series

We create a time series of the probability of adjustment by aggregating the panel using the family weight.
C.3 CEX: Car acquisition probability

We use the consumer expenditure (CEX) survey from 1980-2017 to construct a quarterly time series of the probability of a household acquiring a car or truck (used or new). We use this time series in table 2 to measure the extensive margin response of auto purchases to monetary policy shocks.

We download the pre-compiled files from the BLS for 1996 onwards. The CEX files before 1996 are from ICPSR. We only use the family and expenditure files. We clean the micro-data files following Coibion et al. (2017). We drop households with negative elderly expenditure, zero total expenditure, or zero food consumption. We sum expenditures that occur in the same month but are reported in different interviews as recommended by the BLS. We drop households that report more than three monthly expenditures per interview, since it is difficult to allocate the additional monthly expenditures. We also drop interviews with fewer than three months of data.

In the expenditure files we sum the UCC codes 450110 (new cars), 450210 (new trucks), 460110 (used cars), 460901 (used trucks). All expenditure series are net of trade-in value. This definition aligns with the BEA definition of motor vehicle expenditure. Using the household weights, total motor vehicle expenditure implies by the CEX tracks BEA personal consumption motor vehicle expenditure very well (Figure A.1).

We construct the probability of adjustment by setting an indicator equal to 1 whenever a household’s motor vehicle expenditures are positive, and aggregating the indicator using household weights.

C.4 CPS: Reasons for moving

Reasons for moving are from the March 2001 CPS supplement. We classify these reasons according to whether they broadly fall into life-cycle changes, which more closely correspond to our unmodeled match-quality shocks, and sS-band reasons, which capture the traditional adjustments in fixed cost model from a depreciating durable stock.

We classify wanting a new/better home and cheaper housing is suggestive of durable holdings away from target and price-sensitivity in line with the traditional fixed cost model. These account for roughly a quarter of all adjustments, in line with our calibration of match-quality shocks. Changes in marital status or job-related moving more closely align with life-cycle changes that are less directly tied to interest rates. These account for roughly half of all transitions. The remaining
Figure A.1: Household motor vehicle expenditure from the personal consumption expenditure table in NIPA, and extrapolated from CEX micro-data based on NIPA definitions based on survey weights.

quarter of moves are obviously classifiable in either category.

D Additional Results

D.1 Price levels: non-durables and durables

See Figure A.2.

D.2 Real rate in terms of durables

See Figure A.3.

D.3 Other relative prices: residential investment and cars

See Figure A.4.

D.4 Nominal interest rate responses

Figure A.5 shows the nominal interest rate requires to implement the experiment in which the real interest rate falls by 1% at a horizon of 8 quarters.
Figure A.2: Impulse response function for the level of non-durable prices (left panel) and durable prices (right panel) following a Romer and Romer (2004) monetary policy shock.

Figure A.3: Impulse response function for the real interest rate in terms of durables following a Romer and Romer (2004) monetary policy shock.
Figure A.4: Impulse response function for the price of residential investment relative to the price of non-durables and services (left panel) and the price of motor vehicles relative to the price of non-durables and services (right panel) following a Romer and Romer (2004) monetary policy shock.

Figure A.5: Response of real and nominal interest rates to a change in future interest rates.
Table 5: What was ... main reason for moving?

<table>
<thead>
<tr>
<th>Reason</th>
<th>Frequency</th>
<th>sS-Band</th>
<th>Life-cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in marital status</td>
<td>0.84%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>To establish own household</td>
<td>1.06%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Other family reason</td>
<td>1.92%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>New job or job transfer</td>
<td>1.45%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>To look for work or lost job</td>
<td>0.27%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>To be closer to work</td>
<td>0.44%</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Retired</td>
<td>0.08%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Other job-related reason</td>
<td>0.16%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Wanted to own home, not rent</td>
<td>1.41%</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Wanted new or better house</td>
<td>2.47%</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Wanted better neighborhood</td>
<td>0.55%</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Cheaper housing</td>
<td>0.76%</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Other housing reason</td>
<td>1.52%</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Attend/leave college</td>
<td>0.43%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Change of climate</td>
<td>0.08%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Health reasons</td>
<td>0.19%</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Total</td>
<td>13.64%</td>
<td>3.23%</td>
<td>6.48%</td>
</tr>
</tbody>
</table>

D.5 Forward Guidance in Non-Durables Model

Figure A.6 shows the forward guidance experiment in a version of our model that omits durable goods.

E Details of the General Equilibrium Model

E.1 The Labor Market

The labor demand curve of each labor type $j$ is,

\[ l_{jt} = L_t \left( \frac{W_{jt}}{W_t} \right)^{-\varphi} \]
so that the aggregate wage equal,

\[ W_t = \left( \int_0^1 W_{jt}^{1-\varphi} d\varphi \right)^{\frac{1}{1-\varphi}} \]

The objective of the union is therefore,

\[
\max_{\{\mu_{jt}\}} \int_0^\infty e^{-\rho t} \int_0^1 \left[ u_c(c_{it}, d_{it}) \frac{W_{jt}}{P_t} l_{jt} - \bar{u}_c l v(l_{jt}) - \frac{\Psi}{2} \bar{u}_{c,t} L_t \mu_{jt}^2 \right] d\varphi \, dt
\]

subject to

\[
d\ln W_{jt} = \mu_{jt} \, dt
\]

\[
l_{jt} = L_t \left( \frac{W_{jt}}{W_t} \right)^{-\varphi}
\]

The first order conditions of the union are,

\[
\lambda_{jt} = \Psi \mu_{jt} \bar{u}_{c,t} L_t
\]

\[
d\lambda_{jt} - \rho \lambda_{jt} \, dt = -(1-\varphi) \bar{u}_{c,t} \left( \frac{W_{jt}}{P_t} \right)^{1-\varphi} \left( \frac{W_t}{P_t} \right)^{\varphi} L_t \, dt - \varphi L_t \left( \frac{W_{jt}}{W_t} \right)^{-\varphi} \bar{u}_{c,t} v L(l_{jt}) \, dt
\]

Figure A.6: Response of aggregate consumption to 1% interest rate cut at dates 0 and 8 in model with only non-durable consumption.
Imposing symmetry and market clearing

\[ \Psi \mu_t \bar{u}_{c,t} \frac{Y_t}{Z_t} = \lambda_t \]

\[ d\lambda_t - \rho \lambda_t \, dt = -(1 - \varphi) \bar{u}_{c,t} \frac{Y_t}{Z_t} \, dt - \varphi \bar{u}_{c,t} \frac{Y_t}{Z_t} v_L \left( \frac{Y_t}{Z_t} \right) \, dt \]

\[ \pi_t = \mu_t \]

The non-linear Phillips curve is then,

\[ d\pi_t = \left[ \rho - \frac{d\bar{u}_{c,t}}{\bar{u}_{c,t}} - \frac{dY_t}{Y_t} + \frac{dZ_t}{Z_t} \right] \pi_t \, dt - \frac{\varphi - 1}{\Psi} \left[ \frac{\varphi}{\varphi - 1} v_L \left( \frac{Y_t}{Z_t} \right) - 1 \right] \, dt \]

and log-linearized around the zero inflation state in which \( Y_t = \bar{Y}_t = Z_t v_L^{-1} \left( \frac{\varphi - 1}{\varphi} \right) \),

\[ d\pi_t = \rho \pi_t \, dt - \frac{\varphi}{\Psi} \eta \left( \frac{Y_t - \bar{Y}_t}{Y_t} \right) \, dt \]

where \( \frac{1}{\eta} \) is the Frisch elasticity. Letting \( \kappa = \frac{\varphi \eta}{\Psi} \) gives (10).

### E.2 Market Clearing

Non-durables market clearing:

\[ Y_t = \int_0^1 c_{it} \, di + M_t + G_t + (r^b_t - r_t) \int_0^1 a_{it} I(a_{it} < 0) \, di. \]

Durable goods market clearing:

\[ X_t = \int_0^1 \left( \frac{dd_{it}}{dt} - \delta d_{it} \right) \, di + f \int_0^1 I_{d_{it} \neq d_{it}} \, d_{it} + \nu \int_0^1 d_{it} \, di. \]

Bond market clearing:

\[ \int_0^1 a_{it} \, di = A_t. \]
F Data Filtering Using the MA Representation

Heterogeneous-agent macro models are difficult to represent in a state-space form because the state of the model includes a distribution. For example, in the canonical Krusell-Smith model the distribution of wealth across households is part of the model's state. Because the state-space representation poses a challenge, authors sometimes prefer to work with impulse response functions as an alternative representation of the model's dynamics. These two representations are equivalent for a linear(ized) model (Boppart et al., 2018).

In this appendix we discuss how one can use the impulse response functions of the model to infer the shocks that generated the observed time series data. That is we implement a restricted version of the Kalman filter to recover the shocks. Our principal aim is to recover an estimate of the shocks that generated the data. One can also compute the likelihood of the data. (Auclert et al., 2019) describe an alternative method for computing the likelihood using the MA representation of the model that is not subject to the restrictions we impose. However, their likelihood calculation does not yield estimates of the shocks that generated the data.

We impose two restrictions on the standard Kalman filtering framework. First, we do not allow for measurement error in the observation equation. Second, we assume that either (a) the system is initially in steady state at the start of the sample or (b) the researcher knows the initial state with certainty and knows the transition path of the model back to steady state. If the system is stable, ie all the eigenvalues of the state transition matrix are inside the unit circle, the second restriction is not costly in situations where the researcher has a sufficient burn-in period at the start of the sample so that the effect of the initial state dissipates before the sample of interest begins. The meaning of a ‘sufficient’ burn-in period depends on how persistent the effects of the initial state are.

Typically, researchers use the smoothed estimate of the shocks meaning the estimate of the shock at date $t$ that conditions account of all data observed including data after date $t$. Under the restrictions above, the Kalman filter and the Kalman smoother are equivalent.

Lastly, we describe how to create counterfactuals using alternative shocks and/or alternative impulse response functions.
F.1 Filtering Algorithm

We will present the method starting from a state space representation so that we can compare our method to the Kalman filter.

Consider a dynamic system with a state space representation

\begin{align*}
X_t &= AX_{t-1} + B\epsilon_t \\
Y_t &= CX_t
\end{align*}

where \(X\) is the state, \(\epsilon\) is an i.i.d. mean-zero innovations and \(Y\) is the observed data. \(\epsilon\) and \(Y\) are dimension \(N \times 1\) and \(X\) is dimension \(M \times 1\). \(A\), \(B\), and \(C\) are conformable matrices.

We assume that this internal description of the model is unknown to the researcher. Instead, the researcher has access to an external description of the system, i.e. impulse response functions. Let \(R(\tau,i)\) be the response of \(Y_\tau\) to a unit change in the \(i\)th element of \(\epsilon_0\). The impulse responses are given by

\[ R(\tau,i) = CA^\tau B 1_i, \]

where \(1_i\) is the standard basis vector in the \(i\)th dimension. \(R(\tau,i)\) is a \(N \times 1\) vector. Let \(R(\tau)\) be a \(N \times N\) matrix where the \(i\)th column is \(R(\tau,i)\). Notice that

\[ R(\tau) = CA^\tau B. \]

The researcher may also have access to an estimate of the effects of the initial state of the system \(S(\tau) = CA^{\tau+1}X_{-1}\) for \(\tau \geq 0\). In practice one may wish to assume that the system is initially in steady state so \(S(\tau) = 0\) for all \(\tau\). For a stationary system, where \(A^t \to 0\) as \(t \to \infty\), the role of the initial state will diminish over time so if one has a sufficient burn-in period of data assuming the system starts in steady state will have limited effect on the results.

The researcher has data \(\{Y_t\}_{t=0}^T\) and wishes to recover an estimate of \(\{\epsilon_t\}_{t=0}^T\). The filtering then proceeds recursively as follows: Let \(Q_0 = S(0)\). At date 0 solve (14) and (15) for

\[ \epsilon_0 = (CB)^{-1}(Y_0 - CAX_{-1}) \]

and notice that we can rewrite this as

\[ \epsilon_0 = R(0)^{-1}(Y_0 - Q_0). \]
Now suppose that we have solved for \( \{ \epsilon_\tau \}_{\tau=0}^{t-1} \) and we wish to solve for \( \epsilon_t \). Let \( Q_t = CA \hat{X}_{t-1} \) and by repeated substitution of (14) we have

\[
Q_t = \sum_{\tau=0}^{t-1} CA^{t-\tau} B \epsilon_\tau + S(t)
\]

\[
= \sum_{\tau=0}^{t-1} R(t-\tau) \epsilon_\tau + S(t).
\]

From (14) and (15) we then have

\[
\epsilon_t = R(0)^{-1} \left( Y_t - Q_t \right).
\]

### F.2 Relationship to the Kalman Filter

Let \( \hat{X}_{t|t-1} \) be the point estimate of \( X_t \) given information through \( t-1 \). The Kalman filter updates this estimate as (Hamilton, 1994, eq. 13.2.15)

\[
\hat{X}_{t|t} = \hat{X}_{t|t-1} + P_{t|t-1} C' \left( CP_{t|t-1} C' \right)^{-1} \left( Y_t - C \hat{X}_{t|t-1} \right),
\]

where \( P_{t|t-1} \) is the covariance matrix of \( \hat{X}_{t|t-1} \). Because we assume that the initial state (or rather its effects) is known and there is no measurement error, once \( Y_{t-1} \) is observed, \( \epsilon_{t-1} \) is known and therefore the only reason \( \hat{X}_{t|t-1} \) is uncertain is because of \( \epsilon_t \). Therefore \( P_{t|t-1} = B \Sigma B' \) where \( \Sigma \) is the covariance matrix of \( \epsilon \). Plugging this in above we have

\[
\hat{X}_{t|t} = \hat{X}_{t|t-1} + B \Sigma B' C' \left( B \Sigma B' C' \right)^{-1} \left( Y_t - C \hat{X}_{t|t-1} \right)
\]

\[
= \hat{X}_{t|t-1} + B \left( CB \right)^{-1} \left( Y_t - C \hat{X}_{t|t-1} \right).
\]

Now notice that the update to \( \hat{X}_{t|t-1} \) is just \( B \epsilon_t \) so we have

\[
\epsilon_t = (CB)^{-1} \left( Y_t - C \hat{X}_{t|t-1} \right)
\]

\[
= (CB)^{-1} \left( Y_t - CA \hat{X}_{t-1|t-1} \right)
\]

where the second line follows from Hamilton eq. 13.2.17. Using the logic above, \( X_{t-1} \) is known after \( Y_{t-1} \) is observed so \( \hat{X}_{t-1|t-1} = X_{t-1} \) so the above equation becomes

\[
\epsilon_t = R(0)^{-1} \left( Y_t - Q_t \right)
\]
in the notation of our filtering algorithm, which is the same as (16).

With regard to the Kalman smoother, the smoothed estimate updates the filtered estimate according to (Hamilton eq. 13.6.16)

\[ \hat{X}_{t|T} = \hat{X}_{t|t} + J_t \left( \hat{X}_{t+1|T} - \hat{X}_{t+1|t} \right) \]

where (Hamilton eq. 13.6.11)

\[ J_t = P_{t|t} A' P_{t+1|t}^{-1} \]

In our case there is no uncertainty over \( X_t \) conditional on information through date \( t \) so \( P_{t|t} \) is a zero matrix. Therefore the smoother does not update the filtered estimates of the states and therefore does not update the estimate of the shocks.

**F.3 Counterfactuals**

Now suppose the researcher is interested in some counterfactual outcome. These counterfactuals could come from an alternative history of shocks or an alternative model represented by different impulse response functions. Let \( \{ \hat{Y}_t \} \) denote the counterfactual outcomes, \( \hat{R}(\tau) \) denote the counterfactual impulse response functions, and \( \{ \hat{\epsilon}_t \} \) denote the counterfactual shocks. We then have

\[ \hat{Y}_t = \sum_{\tau=0}^{t} \hat{R}(t-\tau) \hat{\epsilon}_\tau + S(t). \]
G $R^\star$: Definition and Implementation

G.1 Definition

Formally defining $R^\star$ requires us to introduce some general notation. The history of the economy is encapsulated by the (infinitely-dimensional) state vector $x_{t+s|t}$, where the subscript notation is the expected state of the economy at date $t+s$ given information at time $t$. Let $r_{t+s|t} = (r_{t+s|t}, r_{t+s+1|t}, \ldots)'$ be a vector of expected real rates from $t+s$ to infinity given expectations at time $t$. $G$ is the state transition function of the economy such that $x_{t+1|t+1} = G(x_{t|t}, \epsilon_{t+1}, r_{t|t})$, in which $\epsilon$ is a vector of $K$ i.i.d. disturbances. Finally, $Y(x_{t|t}, r_{t|t})$ is a function that gives the output gap given the current state $x_{t|t}$ and the expected sequence of future real rates $r_{t|t}$.

Given a state $x_{t|t}$, we are looking for the sequence of real interest rates $r^\star_t$ that close all future output gaps if there are no future shocks,

$$Y(x_{t+s|t}, r^\star_{t+s|t}) = 0 \quad \forall s \geq 0 \quad (17)$$

$$x_{t+s+1|t} = G(x_{t+s|t}, 0, r^\star_{t+s|t}) \quad \forall s \geq 0 \quad (18)$$

This definition of $r^\star$ is standard in the sense that we are implementing a zero output gap going forward. It is perhaps less standard that the optimal policy explicitly consists of an entire path of real interest rates given the forward-looking nature of household decisions. We will generally report a long-horizon natural rate, $r^\star_{t+h} = \frac{1}{h} \sum_{s=0}^{h} r^\star_{t+s|t}$, to capture future accommodation provided by the central bank.

Our three concepts of $r^\star$ coincide given the same state $x_{t|t}$, but they differ in how the state $x_{t|t}$ evolves.

**Full accommodation:** Under full accommodation, we assume that the central bank in fact implements $r^\star_{t|t}$, and we iterate on the state with that policy,

$$x_{t+1|t+1} = G(x_{t|t}, \epsilon_{t+1}, r^\star_{t|t}).$$

**No accommodation between $[t_0, t_1 - 1]$:** Between $[t_0, t_1 - 1]$ we iterate forward using the full accommodation monetary policy as of $t_0 - 1$,

$$x_{t_0+s+1|t_0+s+1} = G(x_{t_0+s|t_0+s}, \epsilon_{t_0+s+1}, r^\star_{t_0+s|t_0-1}) \quad s = 1, \ldots, t_1 - t_0 - 1$$

65
and at $t_1$ we resume with full accommodation using $x_{t_1}$ as initial state. Comparing the $r^*$ at $t_1$ from full accommodation with the $r^*$ at $t_1$ from no accommodation tells us to what extent full accommodation between $[t_0, t_1 - 1]$ affects $r^*$.

**Actual accommodation:** We iterate the state forward using the observed real rate and its expected path in the data $r_{t|t}$,

$$x_{t+1|t+1} = G(x_{t|t}, \epsilon_{t+1}, r_{t|t}).$$

This concept incorporates incomplete past accommodation in the past was incomplete relative to the natural rate.

### G.2 Implementation

We linearize the system 17-18,

$$\begin{align*}
&Y_x(x_{t+s|t} - \bar{x}) + Y_r(r^*_{t+s|t} - \bar{r}) = 0 \\
&(x_{t+s+1|t} - \bar{x}) = \underbrace{G_x (x_{t+s|t} - \bar{x})}_{\infty \times 1} + \underbrace{G_r (r^*_{t+s|t} - \bar{r})}_{\infty \times 1} \quad (19) \\
&Y_x(G_x)^s(x_{t|t} - \bar{x}) + \sum_{k=0}^{s-1} Y_x(G_x)^{s-k-1} G_r(r^*_{t+k|t} - \bar{r}) + Y_r(r^*_{t+s|t} - \bar{r}) = 0 \quad \forall s \geq 0, \quad (20)
\end{align*}$$

and iterate 20 forward from time $t$,

$$(x_{t+s|t} - \bar{x}) = (G_x)^s(x_{t|t} - \bar{x}) + \sum_{k=0}^{s-1} (G_x)^{s-k-1} G_r(r^*_{t+k|t} - \bar{r}),$$

to obtain a forecast of the state vector given the expected interest rate policy for each future date $t + s$.

Substituting these forecast into the equations for the output gap 19,

$$Y_x(G_x)^s(x_{t|t} - \bar{x}) + \sum_{k=0}^{s-1} Y_x(G_x)^{s-k-1} G_r(r^*_{t+k|t} - \bar{r}) + Y_r(r^*_{t+s|t} - \bar{r}) = 0 \quad \forall s \geq 0,$$

yields a set of equations, one for each future date $t + s$. The first term on the LHS is the IRF of the output gap given the state at time $t + s|t$. Note that this includes the contemporaneous shocks at time $t$. The second and third term yield the IRF of the output gap at time $t + s$ given the real rate path. This includes a direct effect (third term) and effects through the evolution of the state variables (second term). Next, we stack the equations for $s = 0, 1, \ldots$ so that the first row is the
output gap equation at time $t$, the second is the output gap equation at $t+1$ and so on. Combine

terms to get,

$$\begin{align*}
\frac{B_x}{\infty \times \infty} (x_{t|t} - \bar{x}) + \frac{A_r}{\infty \times \infty} (r_{t|t}^* - \bar{r}) &= 0.
\end{align*}$$

The $i,k$ element of $B_x$ is the IRFs of output gap at time $t+i-1$ in response to a deviation the $k^{th}$ state at time $t$. The $i,k$ element of $A_r$ is the IRFs of the output gap at time $t+i-1$ in response to an anticipated change in the real rate $k-1$ periods ahead, $r_{t+k-1}$. The solution to this system is

$$\begin{align*}
(r_{t|t}^* - \bar{r}) &= -(A_r)^{-1} B_x (x_{t|t} - \bar{x}) \quad (21)
\end{align*}$$

However, implementing this solution is infeasible since the state $x_{t|t}$ is unobservable in the data. We next show how we can calculate our $r^*$ concepts using the finite-dimensional shocks $\epsilon_t$, which we extracted using filtering, and an initial condition for $x_{0|0}$ that vanishes in importance through time.

Under full accommodation, the linearized evolution of the state is

$$\begin{align*}
(x_{t+1|t+1} - \bar{x}) &= G_x (x_{t|t} - \bar{x}) + \frac{G_\epsilon}{\infty \times K} \frac{K \times 1}{K \times 1} \epsilon_{t+1} + G_x (r_{t|t}^* - \bar{r})
\end{align*}$$

$$\begin{align*}
&= (x_{t+1|t} - \bar{x}) + G_\epsilon \epsilon_{t+1}
\end{align*}$$

where the second line uses equation (20) with $s = 0$. Therefore next period’s natural rate can be expressed as

$$\begin{align*}
r_{t+1|t+1}^* - \bar{r} &= -(A_r)^{-1} B_x (x_{t+1|t+1} - \bar{x})
\end{align*}$$

$$\begin{align*}
&= -(A_r)^{-1} B_x (x_{t+1|t} - \bar{x}) - (A_r)^{-1} B_x G_\epsilon \epsilon_{t+1}
\end{align*}$$

$$\begin{align*}
&= (r_{t+1|t}^* - \bar{r}) - (A_r)^{-1} C_\epsilon \epsilon_{t+1}
\end{align*}$$

The $i,k$ element of $C_\epsilon$ is the impulse response function of the output gap at time $t+i-1$ in response to a shock to the $k^{th}$ element of $\epsilon$ at time $t$.

Given an initial $r_{0,0}$ and a sequence of shocks $\{\epsilon_t\}_{t \geq 1}$, this equation can be used to update the natural real rate path through time. All that is needed are the IRFs for the output gap to current and future real rate shocks in $A_r$ and the IRFs for the output gap to the contemporaneous shocks $\epsilon$. In practice, we will truncate these matrices at some long horizon $T$ to implement this calculation.
The initial condition \( r_{0,0} \) is unobservable since \( x_{0|0} \) is unknown. However, the importance of this initial condition vanishes as we move forward in time as the initial state \( x_{0|0} \) only has a temporary effect on the output gap and therefore only a temporary effect on the real rate. In other words \( r_{k,0} \approx 0 \) for sufficiently large \( k \). In that sense the choice of the initial state is unimportant if it is set sufficiently far in the past.

To calculate the natural rates under no accommodation and actual accommodation we follow similar steps but condition on different real rate paths. Under no accommodation, the linearized evolution of the state is

\[
\begin{align*}
(x_{t_0+s|t_0+s} - \bar{x}) &= G_x(x_{t_0+s-1|t_0+s-1} - \bar{x}) + G_\epsilon \epsilon t_0 + s + G_r(r^*_{t_0+s-1|t_0-1} - \bar{r}) \\
&= (G_x)^{s+1}(x_{t_0-1|t_0-1} - \bar{x}) + \sum_{k=0}^s (G_x)^k G_\epsilon \epsilon t_0 + s - k + \sum_{k=0}^s (G_x)^k G_r(r^*_{t_0+s-k-1|t_0-1} - \bar{r}) \\
&= (x_{t_0+s+1|t_0-1} - \bar{x}) + \sum_{k=0}^s (G_x)^k G_\epsilon \epsilon t_0 + s - k
\end{align*}
\]

where the last line uses a repeated application of equation (20). The state evolves based on our forecast at the last time when monetary policy accommodated the shocks plus the effect of all the (un-accommodated) shocks that have happened since.

At \( t_1 \) we resume full accommodation as described by the solution (21),

\[
\begin{align*}
r^*_{t_1|t_1} - \bar{r} &= -(A_r)^{-1} B_x(x_{t_1|t_1} - \bar{x}) \\
&= -(A_r)^{-1} B_x(x_{t_1|t_0-1} - \bar{x}) - (A_r)^{-1} B_x \sum_{k=0}^{t_1-t_0} (G_x)^k G_\epsilon \epsilon t_1 - k \\
&= (r^*_{t_1|t_0-1} - \bar{r}) - (A_r)^{-1} \sum_{k=0}^{t_1-t_0} M_k C_\epsilon \epsilon t_1 - k
\end{align*}
\]

The matrix \( B_x(G_x)^k G_\epsilon \) captures the effect of a shock \( k \) periods ago on current and future output gaps. This is equivalent to \( M_k C_\epsilon \) with \( M_k = \begin{pmatrix} 0_k & I_\infty \end{pmatrix} \) an \( \infty \times \infty \) block matrix containing zeros in the first \( k \) columns followed by an identity matrix. The initial natural rate path \( r^*_{t_1|t_0-1} \) is a solution to the full accommodation problem and can be obtained following those steps. Similarly, all natural rates after \( t_1 \) can be obtained by using \( r^*_{t_1+1|t_1} \) as an initial natural rate path in the full accommodation solution.

Under actual accommodation, the evolution of the state is now governed by realized and ex-
pected monetary policy,

\[
(x_{t+1|t+1} - \bar{x}) = G_x(x_{t|t} - \bar{x}) + G_x\epsilon_{t+1} + G_r(r_{t|t} - \bar{r})
\]

\[
= (x_{t+1|t} - \bar{x}) + G_x\epsilon_{t+1}
\]  \hspace{1cm} (22)

and solves

\[
r_{t+1|t+1} - \bar{r} = -(A_r)^{-1}B_x(x_{t+1|t+1} - \bar{x}) + (A_r)^{-1}Y_{t+1|t+1}
\]

where \(Y_{t+s|t} = (Y_{t+s|t}, Y_{t+s+1|t}, \ldots)\) are the output gaps starting at time \(t + s\) under the expected monetary policy at time \(t\). Taking expectations as of time \(t\), we get

\[
r_{t+1|t} - \bar{r} = -(A_r)^{-1}B_x(x_{t+1|t} - \bar{x}) + (A_r)^{-1}Y_{t+1|t}
\]  \hspace{1cm} (23)

Note that when the current policy closes all future output gaps, \(Y_{t+s|t} = 0\), then (23) coincides with (21).

Next periods \(r\)-star can again be derived using equation (21),

\[
r^*_t - \bar{r} = -(A_r)^{-1}B_x(x_{t+1|t} - \bar{x})
\]

\[
= -(A_r)^{-1}B_x(x_{t+1|t} - \bar{x}) - (A_r)^{-1}C\epsilon_{t+1}
\]

\[
= (r_{t+1|t} - \bar{r}) - (A_r)^{-1}Y_{t+1|t} - (A_r)^{-1}C\epsilon_{t+1}
\]

Line 2 uses the state transition (22), and line 3 uses the solution for the expected real interest rate at time \(t\) (23).

\section{Model Impulse Response Functions}

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Impulse} & \textbf{Response} & \textbf{Description} \\
\hline
\textit{\(r\)} & \(\Delta G_x\) & \(\Delta \bar{x}\) \\
\hline
\textit{\(\Delta G_x\)} & \(\Delta \bar{x}\) & \(\Delta \bar{r}\) \\
\hline
\end{tabular}
\end{table}
Figure A.7: Impulse response functions for the output gap $\hat{Y}$, the change in the durable expenditure share relative to potential GDP $\Delta s^x$, the real interest rate $r$, the borrowing spread $r^b$, and the contemporaneous natural rate of interest $r^*$ following a shock to productivity $e^Z$, non-household demand $e^G$, the monetary policy rule $e^r$, and the borrowing spread $e^{rb}$. 