Measuring Brevity and Violence: A Comparison of Approaches

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1 Introduction

McKay and Reis (2008) presented a new business cycle fact—contractions in employment are briefer and more violent than expansions of employment while contractions of output are not significantly briefer or more violent—and to explain it offered a model of business cycles with adjustment costs of employment and an active choice of when to adopt new technologies. In this companion note, we complement their results in two ways:

(i) McKay and Reis (2008) presented results for one pair of employment and output series and using a single method to detrend the data and a single method to detect turning points. Here, we systematically explore other data series and methods to show that the new fact is robust. We first discuss various alternative methods that can be employed for detrending the data, detect turning points and making inference about brevity and violence across expansions and contractions. We then present the results of employing those different methods and other checks on our procedure.

(ii) McKay and Reis (2008) presented the model and stated a series of propositions that describe its properties. Here, we prove these propositions, and describe an algorithm that numerically solves the model.

2 Is the fact robust? Methods

This section discusses alternative methods to infer the brevity and violence of contractions and expansions. First, we discuss why the statistics we use are informative. Then, we discuss the need to de-trend and different methods to do so. We then present alternatives to detect peaks and troughs and compare them with the NBER chronology. Finally, we discuss how to measure brevity and violence and how to statistically infer if they are the same in expansions and contractions. The following section applies some of these alternative methods to the U.S. data to assess the robustness of our results.

2.1 Statistical approach to business cycle facts

Our empirical strategy requires five steps: (i) choosing a measure of business activity, (ii) using an algorithm that de-trends it, (iii) picking turning points, (iv) measuring brevity and violence, and (v) systematically comparing them.[1] These steps are familiar to the literature on business-cycle facts.[2] What we are departing from is the dominant tradition in the last 20 years of looking at second-order moments of de-trended data. The question that we want to answer requires contrasting expansions to contractions and looking for asymmetries in brevity and violence. This cannot be done with second moments, but requires instead turning-point algorithms and measures of brevity and violence.

Also in common with the large empirical business-cycle literature surveyed in Stock and Watson (1999), we try to identify a pattern in the data using simple statistics. Our aim is to find a robust pattern in the data, that can be used to suggest models of economic behavior. Hopefully, this will lead in the future to settling on a particular model that can then be used to examine more specific features of the data. But to start, one needs a robust fact that imposes as little structure as possible. We adopt a statistical approach for this reason.

2.2 Trends and de-trending

If output trends up, only in rare instances where it is hit by an unusual sequence of negative shocks will it ever decline. The positive trend automatically leads to longer expansions and shorter contractions. The question of whether "contractions are briefer and more violent than expansions" is simply not interesting in trending data; the answer is yes, by definition. In other words, the business cycle facts on brevity and violence refer to "growth cycles" as opposed to "classical cycles" (e.g., Zarnowitz, 1992).[3] To address the question requires de-trending the data.[4]

[1] Looking at the skewness of 4-quarter changes gives us an alternative test that does not require steps (ii) and (iii).

[2] Take, for instance the survey of Stock and Watson (1999) where, among others, they investigate whether "employment is pro-cyclical but lags output." To see if this is true they take the five steps by: choosing measures of employment and output, using a band-pass filter to de-trend them, refering to booms and recessions as periods where the series are above or below their trend, computing co-movement via cross-correlations at different lags and focusing on the contemporaneous correlation and its one-period-lagged counterpart, and inferring that employment is procyclical if the contemporaneous correlation is sufficiently high and that it lags output if the 1-period lag correlation is higher. For a more recent example of this approach, see Harding and Pagan (2002).

[3] An analogy may be useful to understand this point. Investigating the well-established facts: "consumption is more stable than income" or "investment is more volatile than output" also cannot be done with trending data. All of these series trend up, so their sample variances on any finite sample are completely uninformative on these facts. One must de-trended the data to investigate them.

[4] Fixing the dates of peaks and troughs (and so the fact on brevity), we can use trending data to investigate the fact on violence. Unsurprisingly, given the robustness of our facts to de-trending procedure, doing this confirms our conclusions.

Another reason to de-trend the data is that an increase in trend growth automatically leads to rarer contractions with smaller declines in the raw series. In this case, brevity and violence become a feature of the trend, and not just of the cycle. If one takes the view that trend and cycle can be studied separately (an arguable but popular position) then it would be undesirable to study a business cycle property that depends on trend growth.

The case for de-trending unemployment is less clear cut. We chose to de-trend it for three reasons. First, because even though the unemployment rate does not trend up or down, it has a significant low frequency component driven by demographic changes. Using the raw series can lead to misleadingly observing very short or very long business cycle phases around the time of changes in this component. Second, we want to ensure that the differences between employment and output are not caused by treating the series differently. Third, when we experimented using the raw data, we found little difference from the results with de-trended data. Removing trends seemed not to matter too much for unemployment, so for consistency and comparability with the output results we chose to de-trend it as well.

One difficulty with de-trending is that there is no consensus on what is the best way to do it. To investigate the robustness of our results, we take an eclectic approach using four algorithms that broadly capture four different views of the source of trends. The first view sees trends as deterministic but subject to occasional abrupt changes in growth rates. To represent this view, we fit a linear regression of time, allowing for breaks in the slope in 1973:4 and 1995:4 to capture the productivity slowdown.[5] The second view agrees that the trend is deterministic, but models changes that occur smoothly. We fit a polynomial function of time to the series, using measures of goodness of fit to pick the order of the polynomial. The third view associates trends with possibly stochastic movements affecting the low frequency of a series. We use the Baxter and King (1999) band-pass filter to extract cycles of duration between 6 and 32 quarters for output and between 2 and 80 quarters for employment. These are the common choices in the literatures on output and employment respectively, and section 3.2 will consider other choices for duration. A fourth view of the trend insists that it should be smooth and uncorrelated with the cycle. We calculate it using the minimization algorithm of Rotemberg (1999), which builds on the Hodrick-Prescott procedure but sets the smoothing parameter to the lowest possible value that

[5] We experimented with close alternatives dates for the breaks and found no substantial differences.

ensures that changes in the trend are uncorrelated with the cycle.[6] While we obtained very similar results using the Hodrick-Prescott filter, we found that the Rotemberg alternative performed better at the edges of the sample.[7]

2.3 Detecting expansions and contractions and the NBER

Expansions and contractions are defined by peaks and troughs. The peak marks the end of an expansion and the beginning of a contraction, while the trough marks the end of a contraction and the beginning of an expansion. To investigate the robustness of our results, we consider four different algorithms to detect turning points that are broadly representative of the available menu.[8] The appendix describes each method in more detail.

The first method, which we label the window method, searches for local extremes. It starts by smoothing the series using a 5-quarter centered moving average to remove high-frequency noise. Then, at each date, it forms a symmetric window with N quarters around each side of the date. If the date is a maximum (minimum) in the window, then it becomes a candidate peak (trough). Finally, to ensure that peaks and troughs alternate, we take the later of two consecutive peaks (troughs). We set $N = 5$, since it leads to a number of cycles not too different from that of the NBER, but the results are similar if N is 3 or 7.

Second is the reversal method, which looks for reversals in the successive changes in the series. This method finds peaks at dates which are preceded by N periods of successive increases and $N - 1$ quarters of successive decreases. Troughs are dates preceded by N decreases and followed by $N - 1$ increases. This method captures the often-held view that a contraction is a period of some quarters of negative growth. We chose $N = 3$, for the same reasons as in the window method.

[6] More precisely, given a series $\{x_t\}_{t=1}^{T}$, the modified-HP filter solves:

$$\min_{\{\bar{x}_t\}_{t=1}^{T}} \sum_{t=2}^{T-1} \left[(\bar{x}_t - \bar{x}_{t-1}) - (\bar{x}_{t-1} - \bar{x}_{t-2})\right]^2 + (1/\lambda) \sum_{t=1+k}^{T} (x_t - \bar{x}_t)(x_{t-k} - \bar{x}_{t-k})$$

to obtain a trend series as a function of λ, $\bar{x}_t(\lambda)$, and then picks the optimal λ^* by solving:

$$\lambda^* = \min_{\lambda} \left\{ \lambda : \sum_{t=k+v}^{T-k-v} (x_t - \bar{x}_t(\lambda)) \left[(\bar{x}_{t+v} - \bar{x}_t) - (\bar{x}_t - \bar{x}_{t-v})\right] = 0 \right\}.$$

where $k = 16$ and $v = 5$.

[7] Missing from our list of de-trending algorithms are unobserved-components models. We avoided these because they impose tight statistical restrictions on the series that affect their symmetry.

[8] For a discussion of alternative methods to pick turning points, see Canova (1999), Harding and Pagan (2002), and Zarnowitz and Ozyildirim (2002).

Third is the Bry and Boschan (1971) approach, which is a more refined version of the window method. Bry and Boschan found that their algorithm reproduces the set of turning points picked by Burns and Mitchell (1946) and the NBER, and King and Plosser (1994) and Watson (1994) have confirmed this finding. While the exact algorithm contains several steps, it can be broadly described as follows. First, the algorithm smooths the series using a 1-year centered moving average and looks for peaks and troughs in a manner akin to the window method. Second, it smooths the series using an alternative moving average (a Spencer filter) that allows it to have sharper changes, and again looks for turning points. Third, it looks for turning points in a shorter (3-month) centered moving average. Finally, the algorithm looks for peaks and troughs in the unsmoothed series using a series of criteria to eliminate mistakes that may be caused by erratic movements.

Our fourth algorithm is in the Markov regime-switching tradition and is due to Chauvet and Hamilton (2005). It assumes that a series $x(t)$ alternates between two states, so either $x(t) = x_1(t)$ or $x(t) = x_2(t)$. The state is a latent variable that follows a first-order Markov chain where the probability of staying in state 1 is p_1 and the probability of staying in state 2 is p_2. In each state, $\Delta x_1(t) \sim N(\mu_1, \sigma_1^2)$ and $\Delta x_2(t) \sim N(\mu_2, \sigma_2^2)$. Associated with this statistical model is a likelihood function that we can maximize to find estimates of the six parameters $(p_1, p_2, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$. Note that we use this approach, not as a model of the stochastic process, but solely as an algorithm to provide a statistic of the sample path. It provides estimates at each date of the probability of being in either of the two states, that we use to define expansions as the dates when the probability of being in the high-mean state is higher than 50%, and contractions when it falls below 50%. In practice the estimated probability is above 80% and below 20% most of the time, so the results are not sensitive to the 50%-cutoff rule. An important caveat to this approach is that it does not impose that the two states correspond to expansions and contractions. Indeed, when we use this algorithm, we find that the two states corresponded to pre and post 1984, marking the fall in output volatility that has been called the great moderation. We extend the model to allow for σ_1^2 and σ_2^2 to differ pre and post 1984:3, raising the number of parameters to eight.[9] While the states identified by the algorithm then more closely resemble expansions

[9] We have looked at a few quarters before and after this exact date and obtained similar results. A less appealing alternative is to impose $\sigma_1^2 = \sigma_2^2$, since it constrains differences in violence solely to differences in μ_i. When we tried this alternative, the resulting turning points were quite similar.

and contractions, one should still keep this caveat in mind.

The first three methods tend to produce similar dates for peaks and troughs, while the last one typically detects fewer business cycle and thus longer contractions and expansions. All of the methods provide a different set of dates from the NBER's. Figure 1 shows this by plotting also the NBER dates. Why are they different? Since the NBER subjectively looks at many series without committing to any particular method, it is impossible to answer this question definitely. Still, the original motivation for Bry and Boschan's (1971) work was precisely to provide a formal counterpart to the NBER's decisions. They found that their algorithm could closely match the NBER dates.

Using the Bry-Boschan algorithm on GDP until 2005, we can reproduce almost exactly the NBER dates until now, as long as we do not de-trend the data. The difference between our dates and the NBER's seems to boil down to the issue of de-trending. As we explained earlier, trending data cannot say anything meaningful about the brevity and violence of contractions and expansions. Therefore, the NBER dates are not appropriate for our investigation, even if they are useful for many other purposes.[10]

2.4 Measuring brevity and violence

Measuring brevity is straightforward: since the duration of a contraction (expansion) is the number of periods from peak to trough (trough to peak), brevity is simply understood as smaller duration.

Violence is the rate of change of the series $x(t)$, or how quickly it falls and rises. In the main paper, we measured it simply by the (absolute value of the) average change in the series. From now onwards we call this particular measure of violence, steepness.[11] As an alternative, we consider another measure of violence, the square root of the average squared change in $x(t)$, which we call sharpness. It is easy to show that $sharpness^2 = steepness^2 + VAR(\Delta x(t))$, so a sharper contraction is either steep or has the series jerking around by more. Our third measure, which we call slope, is the least-squares coefficient on

[10] The NBER itself has not always been consistent about whether to de-trend the data or not. While post 1927, it has focussed on trending data, Romer (1994) convincingly shows that the business cycle dates for the 1884-1927 period came from looking at de-trended data. This is consistent with Mitchell's own view, which seems to have hesitated between de-trending or not, as discussed by Romer.

[11] For a de-trended series, the numerator in steepness (the total change in the series from one turning point to the next) must on average be the same for expansions and contractions. Since the denominator in steepness on average equals duration, then a brief series will tend to be a violent series as well, although not necessarily so.

a linear trend from a regression of $x(t)$ on the trend and an intercept. Its virtue is that it is less sensitive to the exact location of peaks and troughs, which we are likely measuring with some error.

If during a contraction (or expansion) a series falls exactly linearly, then *steepness* = *sharpness* = *slope*. Otherwise, sharpness adds to steepness a measure of how volatile the series is, while slope makes the measure of violence less dependent on the exact location of the turning points. None of these measures is "right" in a well-defined sense. They are all just statistics that are trying to capture a feature of the data, violence, in the same way that the standard deviation, the interquartile range or the mean absolute deviation are different statistics to measure volatility.

2.5 Statistical inference

The algorithms discussed so far produce a set of measures of duration and violence $\left\{D_x^S(i,p), V_x^S(i,p)\right\}$, indexed by the series used ($x = y, e$), whether we are in an expansion or contraction ($S = E, C$), the cycle within a series in the sample, ($i = 1, ...I$), and the procedure used to transform the data, to identify turning points, and to measure violence (p). We employed four different approaches to infer whether contractions are different from expansions.

First, we looked at the cumulative distribution functions (cdf's) across i to see whether the cdf for the duration of contractions tends to lie to the left of the cdf for expansions, and the reverse for violence. Plotting these allows us to graphically infer whether contractions tend to be briefer and more violent than expansions. At the extreme, if $F(D_x^C(.,p)) \geq F(D_x^E(.,p))$ and $F(V_x^C(.,p)) \leq F(V_x^E(.,p))$, then the duration of expansions first-order stochastically dominates the duration of contractions, while the opposite is true of violence.

Second, we tested the null hypotheses of equal average duration $E[D_x^E(.,p)] = E[D_x^C(.,p)]$ against the one-sided alternative of shorter contractions $E[D_x^C(.,p)] < E[D_x^E(.,p)]$, and equal violence $E[V_x^E(.,p)] = E[V_x^C(.,p)]$ against more violence in contractions $E[V_x^C(.,p)] > E[V_x^E(.,p)]$. If duration and violence are independent over i, then a standard t-test of equality of means is efficient in a finite sample under normality, and asymptotically efficient otherwise. The assumption of independence may be problematic, so in section 3.2 we use a bootstrap to produce distributions for the t-statistic when the duration and violence are correlated across successive cycles.

Third, we tested the null hypothesis that output is as asymmetric as employment.

Specifically, letting $x(p) = \sum_{i=1}^{I}[D_e^E(i,p) - D_e^C(i,p)]/I$ for employment, we tested whether $E[D_y^E(.,p)] - E[D_y^C(.,p)] = x(p)$ against the one-sided alternative that the difference in average duration in expansions and contractions for output is smaller than that for employment. Likewise, for violence, we computed $z(p) = \sum_{i=1}^{I} V_e^E(i,p)/\sum_{i=1}^{I} V_e^C(i,p)$ and tested whether $E[V_y^E(.,p)]/E[V_y^C(.,p)] = z(p)$. We use ratios, rather than differences, to adjust for different units since output is more volatile than employment.

Fourth, we computed the skewness of $x(t) - x(t-4)$ for each series and performed a standard asymptotically-normal test of whether it is equal to zero. We use the test statistic and significance values of Bai and Ng (2005), which are robust to the serial correlation in the data. Aside from applying this test to the raw series, as in the main paper, we also do it with regards to the de-trended series, as a joint check on both the method of de-trending and the test for skewness.

We also consider a fifth approach. We test the null hypothesis that the distributions from which duration and violence are drawn are the same for expansions and contractions using a Wilcoxon rank-sum test and computing the exact p-values for each sample size. Diebold and Rudebusch (1992) note that this test can be quite efficient even in small samples. It also requires the assumption of independent draws, so we again employ the bootstrap to calculate its distribution if there is serial correlation.

One possible criticism of these tests is that they treat the $D(.)$ and the $V(.)$ as observations, even though these are the product of the algorithms that we described so far. We do not think that this is a matter of too much concern. Most macroeconomic series, like output or consumption, are also the result of algorithms with many steps that add, subtract, average, interpolate and smooth. Since our algorithms are symmetric, they do not create any asymmetry between expansions and contractions beyond the one already in the data. Still, we address this concern in section 3.2.4 by using estimated symmetric models to generate artificial times-series of the same length as our sample on which we apply our algorithms and tests and check whether we could reach erroneous conclusions.

3 Is the fact robust? Results

For the majority of this section, we measure output using the log of industrial production (IP) and employment using the log of one minus the unemployment rate at a quarterly frequency from 1948:1 to 2005:1.[12] For the main results on robustness, we consider 16 methods for duration (4 for de-trending and 4 for detecting turning points) and 48 for violence (plus 3 for measuring violence).

3.1 Main results

Figure 1 plots the cdf's for the duration of unemployment. In almost all cases, the distribution during expansions either strictly stochastically dominates that for contractions or almost always lies to the right of it. In contrast, figure 2 plots the cdf's of brevity for output. The distributions typically lie on top of each other without a discernible difference between expansions and contractions, aside from the presence of a single expansion that was longer than usual.

Tables 1a to 1d present the average duration of expansions and contractions, as well as the t and W statistics and the respective p-values for the tests of equal means and equal distributions. Across the different methods, the average length of an expansion in unemployment is about 16 quarters, whereas the average length of a contraction is only about 7 quarters long. The difference is significant at the 5% level for most cases, and at the 1% level for many of them. For output however, the average length of an expansion is about 11 quarters, whereas the average contraction is about 8 quarters long. In most cases, the difference is not statistically significant at the 5% level. Moreover, if one excludes the single long expansion that stood out in figure 2, the difference falls to below 1 quarter. The null hypothesis that the difference in the duration of expansions and contractions in output is as large as that for output is typically rejected at conventional significance levels. Therefore, with regards to duration, the data strongly suggests that contractions are briefer than expansions in employment by about 9 quarters. For output, the difference between expansions and contractions is much smaller, 2 or 3 quarters, and we cannot reject the hypothesis that it is zero. This confirms results 1 and 2 regarding brevity.

To understand what lies behind the difference in brevity, within each method, we compared the dates at which peaks and troughs occur in output and in employment. The typical finding is that peaks in employment lag peaks in output by between 1 and 3 quarters, whereas troughs in employment are typically within one quarter of troughs in output.

[12] Using GDP instead of IP leads to almost exactly the same results. We prefer to use IP in this section because it is also available at a monthly frequency, allowing for further robustness checks.

The brevity in the contractions in employment is due to employment starting to decline only after output has already been declining for some time. The contractions in both output and employment end around the same time, confirming result 3.

Turning to violence, figures 3a to 3c show the cdf's for the three measures of violence in employment, and figures 4a to 4c in output. The contrast between the two is clear. Whereas contractions are substantially more violent than expansions for employment, the cdf's for output are typically very close, except when using the Chauvet-Hamilton algorithm. Tables 2a to 2d show the results from the t and W tests. For employment, most tests (89 out of 96) reject the null of symmetry at the 5% level. For output, we fail to reject the null hypotheses of equal violence at the 5% significance level only in 13 cases, and at the 1% level only once. All of the rejections for output occur when the Chauvet-Hamilton algorithm for picking turning points is used. Testing the null hypothesis that the difference in output is as extreme as that in employment leads to p-values typically around 0.07. Therefore results 1 and 2 are also robust with regards to violence.

Finally, we calculate the skewness of 4-quarter changes in IP and the employment rate for the de-trended series using each of the 4 methods of de-trending. Using piecewise-linear, polynomial, band-pass and modified-HP filtering, the skewness coefficients and p-values for a one-sided test of zero skewness for output are respectively -0.31 (0.18), -0.47 (0.07), -0.24 (0.26) and -0.23 (0.26). We can never reject zero skewness at the 5% significance level. For employment, they are -0.61 (0.03), -0.62 (0.03), -0.54 (0.04) and -0.50 (0.06), rejecting the null of zero skewness at the 5% level zero in 3 out of the 4 cases.

To conclude, while the results with regards to violence are not as overwhelming as with regards to brevity, the evidence strongly supports the view that contractions in employment are more violent than expansions. For output instead, we typically cannot reject the hypothesis of equal violence in expansions and contraction, but there is some evidence that the difference is not as strong as that for employment.

3.2 Further robustness

We now summarize additional results using other data series and checks on our original results.

3.2.1 The frequency of the observations

One may fear that quarterly data might not be fine enough to accurately detect turning points. It is unclear that this would bias our measures of brevity and violence in a particular direction, or that it would do so differently for output and employment. Still, we check this using monthly seasonally-adjusted observations for industrial production and the unemployment rate and present those results in table 3. As before, there are very few rejections of the null that expansions and contractions in output are equally long and violent. For employment, the results are not as strong as with quarterly data, but one still rejects symmetry for the majority of cases.

3.2.2 The series used

When using the band-pass filter we used parameters (6,32) for output and (2,80) for employment. Using (2,80) for output does not change the results in table 2c. Using (6,32) for employment typically raises the p-values leaves the inferences for brevity unchanged, but there is less evidence of violence. These results are presented in tables 1e and 2e.

Perhaps there is something special about the series for industrial production and the unemployment rate. Turning first to output, we consider also GDP and non-farm business output to ensure that our results are not driven by some specific features of the industrial sector. As a second check, we see whether inventories or indirect taxes and depreciation may enhance or abate asymmetries by considering series for real sales and real personal income. As a third check, we break output into consumption, investment and government spending and look for asymmetries in these series. The results appear in table 4 and the basic statistical inference is unchanged: we cannot reject the null that contractions and expansions are equally brief and violent.

We have also compared the timing of peaks and troughs across the different output series. With the exception of government expenditures, the dates are typically similar, within 2 quarters of each other in most cases. This is reassuring on two accounts. First, it gives us some confidence that our dating of turning points correctly identifies the business cycle in output. Second, it suggests that looking at many series at the same time in a multivariate approach to detect turning points would lead to similar results to our univariate approach.

Still on output, we looked also at a series from another time period: monthly pig-iron production between 1877 and 1929 and present results in table 5. If one looks solely at the t-test for same average duration, then there is evidence for shorter contractions than expansions in this output series that Mitchell and others focussed on. However, looking at either the Wilcoxon test or at any of the measures of violence, we cannot reject symmetry.

Looking next at employment, we separate the labor force from total employment by looking at the total number of employed according to the household survey. Results appear in table 6. Contractions in total employment are still typically briefer and more violent than expansions, although p-values are a little higher. Using instead payroll employment from the establishment survey, there is stronger evidence of asymmetry.

Next, we look separately at the employment rate for younger and older workers. The evidence of briefer and more violent contractions is stronger among workers over 24 than it is for workers between 16 and 24, but it is present for both. Another labor market variable that attracts attention is the participation rate. When we applied our algorithms though, we found that we could typically not find that many turning points.

Finally, we looked at hours. Total hours behave in a similar way to output, with similar dates for turning points and similar estimates of duration and relative violence. Hours per worker are different. Most often contractions and expansions are equally brief and violent for hours per worker, but the results are less clear-cut. Moreover, its turning points are often quite different than those found for employment or output. It seems that cycles in hours per worker do not resemble cycles in output or employment.

3.2.3 The turning-point algorithms

While we have already ensured some robustness to the particular algorithm for finding turning points by using 4 alternatives, we explore further the specifics of each algorithm.

A first concern arises with the window algorithm. When it identifies two successive candidate peaks (or troughs), we took the latter. Our reasoning was that, during expansions, the series may have very short-lived blips downward that lead to incorrectly detecting a peak there. The reverse reasoning applies to contractions. We also tried an alternative selection rule, that takes the higher of the two candidate peaks. We found that the dates of turning points were almost entirely unchanged.

Second, we implemented a simple and effective test of whether there is some hidden feature in the algorithms that causes asymmetries. Taking each series, we reversed its timeordering and ran our algorithms. Looking from the perspective of the present in the direction of the past, expansions now become contractions and contractions become expansions. We found that the algorithms pick out the same turning point dates in 87% of the cases, with the failures evenly distributed between peaks and troughs.

Yet a third strategy to check whether the algorithms are doing the right job is to simulate artificial data and see whether the right turning points are detected. As a data-generating process we use a Chauvet-Hamilton model with parameters that imply symmetric expansions and contractions. We simulated 1000 samples of the same length as our data, and ran our turning-point algorithms on each, recording whether they detected turning point at the right dates. We found that all four of our methods to detect turning points have a close to 100% success rate, as long as the preceding expansion (or contraction) lasts for more than 2 quarters.

A fourth concern might be that the difference that we find between output and employment is driven by finding many short and symmetric cycles for output and only a few and very asymmetric cycles for employment. We checked if this was the case by computing the difference between the number of cycles in output and in employment. The average difference across the 16 classifications was 1.2, so the algorithms are detecting approximately the same number of cycles in output and employment, and excluding the extra cycles in output, does not alter the results.

3.2.4 The statistical tests and inference

A first concern with statistical inference is that our tests are based on small samples of expansions and contractions, typically around 23, and use the assumption of independent draws of duration and violence. We address this using one Monte Carlo experiment that allows for serial correlation.

We allow for the possibility that longer expansions are followed by shorter (or longer) contractions, by estimating an AR(1) on the sequence of durations for contractions and expansions demeaned by their group averages. We then simulated artificial samples of data using the estimated autoregressive coefficients but setting the intercept to ensure that the mean duration of both recessions and contractions was the same. Using this symmetric data generating process, we draw innovations from a normal distribution to generate 23 observations. We then run our algorithms and construct the t and W statistics. Repeating this 1000 times generates an empirical distribution for these statistics, under the assumption of symmetry but now allowing for serial correlation. We found that the bootstrap p-values for the test of symmetry in duration or violence were close to their asymptotic counterparts, only slightly more conservative. For the majority of cases, as before we reject symmetry for employment but do not reject it for output.

A second concern with our tests is that we treat the $D(.)$ and the $V(.)$ as observations. Insofar as these are measured with error, the standard errors used for our tests may underestimate the sampling error. We conduct a second Monte Carlo exercise to investigate this issue. We use a Chauvet-Hamilton model as a data-generating process, with the Markov transition probabilities set so that the average duration of contractions and expansions is 10 quarters for both output and unemployment. We allow the variances to change pre and post 1984, but impose that the changes in the series in the two states have the same mean and variance. These are then estimated for output and unemployment separately. Using this symmetric data generating process, we simulate 1000 samples for output and unemployment. Treating these as data, for each sample we run our algorithm to detect turning points and construct the t and W statistics. The results are a little surprising. It turns out that the p-values are typically lower than before. The rejections of symmetry for employment are stronger than before, whereas for output, one can still typically not reject symmetry at least at the 5% level. The exception is when the Chauvet-Hamilton algorithm to detect turning points is used, in which case the p-values increase considerably.

3.2.5 An alternative approach to statistical inference

An alternative approach is to commit to a statistical model that completely characterizes the observations of output and employment and allows for, but does not require, asymmetries between expansions and contractions. With this model in hand, one can test for the symmetric case as nested in the general specification.

Our model of the data is the version of the Chauvet-Hamilton model described in the previous section. Whereas in that section, the model was treated as an algorithm to detect turning points, here it is treated as a full statistical representation of the data on output and employment. We estimate the parameters $(p_1, p_2, \mu_1, \mu_2, \sigma^2_{1,pre}, \sigma^2_{2,pre}, \sigma^2_{1,post}, \sigma^2_{2,post})$ by maximum likelihood and build likelihood-ratio tests for the null hypotheses of equal brevity: $p_1 = p_2$, and equal violence, either measured as equal steepness: $\mu_1 = \mu_2$, or equal sharpness: $\mu_1 = \mu_2$, $\sigma^2_{1,pre} = \sigma^2_{2,pre}$, and $\sigma^2_{1,post} = \sigma^2_{2,post}$. The results are in table 7. Typically, we could not reject the hypothesis that contractions are as long as expansions for both output and employment. As for violence, for output, at the 5% significance level, we cannot reject the hypothesis of equal steepness but reject equal sharpness. For employment, we reject equal violence for both measures of violence at the 5% level.

3.3 The bottom line

After trying hundreds of different combinations of the available methods and looking into the details of how each works, we found that our main results are very robust. The pattern that emerges from the data is clear: contractions in employment are briefer and more violent than expansions. Contractions and expansions in output are either equally brief and violent or slightly different, but definitely less asymmetric than employment. Employment and output differ because employment typically lags output at peaks but they roughly coincide in their troughs.

4 The model

McKay and Reis (2008) stated a series of propositions describing their model. Here, we present the proofs of their propositions, referring the reader to that paper for the notation and the statement of the results.

4.1 Proof of Proposition 1

Every period, the firm allocates workers to tasks. Obviously, it will fill the most productive tasks first, so there is a threshold x_t such that only tasks with $A_j > x_t$ are operated. The number of workers is then $N_t = 1 - \Phi^E(x_t)$ or $N_t = 1 - \Phi^N(x_t)$.

Given the assumptions, at stage 2, the representative consumer solves the static problem:

$$\max_{L_t, x_t, \{q_{j,t}, l_{j,t}\}} \left\{ \alpha\gamma t + \alpha \ln L_t - b \int_x^{\bar{A}} (q_{j,t} + l_{j,t})\, d\Phi^E(A_j) \right\} \tag{1}$$

$$s.t. \quad : \quad L_t = \int_{x_t}^{\bar{A}} A_t(A_j q_{j,t} + l_{j,t} - z) d\Phi^E(A_j) - \kappa, \tag{2}$$

$$0 \quad \leq \quad q_{j,t} \leq 1, \;\; l_{j,t} \geq 0. \tag{3}$$

Letting λ denote the Lagrange multiplier on the labor input constraint, which will measure the marginal product of labor, the first-order conditions are:

$$(L_t) \quad : \quad \alpha/L_t = \lambda \tag{4}$$

$$(l_{j,t}) \quad : \quad \lambda A_t \leq b \text{ and } l_{j,t} \geq 0 \text{ w.a.l.o.e.} \tag{5}$$

$$(q_{j,t}) \quad : \quad \lambda A_t A_j \geq b \text{ and } q_{j,t} \leq 1 \text{ w.a.l.o.e.} \tag{6}$$

$$(x_t) \quad : \quad b\,(q_x + l_x) = \lambda A_t (x q_x + l_x - z), \tag{7}$$

where w.a.l.o.e. is "with at least one equality." Since $A_j \geq 1$, comparing the first-order conditions for $l_{j,t}$ and $q_{j,t}$, if any worker works overtime, then all workers must be working full time. Therefore, $q_{j,t} = 1$ and $b/\lambda = A_t$. The first-order condition for the number of workers x_t then implies, after some rearranging, that $x_t = 1 + z$. This proves part b) of the proposition. (This leaves open the case where $l_{j,t} = 0$ for all. Then, the first-order condition for $q_{j,t}$ implies that $q_{j,t} = 1$ for $j \in (x, \bar{A}]$, but $q_{x,t} < 1$ and $\lambda A_t A_x = b$. The first-order condition for x_t shows that this cannot be an optimum—at this low marginal product of labor, it is not worth keeping the worker.)

The first-order condition for L_t implies that $L_t = \alpha A_t / b$. Plugging this into the production function $Y_t = (e^{\gamma_t} L_t)^{\alpha}$, gives the expression in part a) of the proposition.

Since overtime is equally productive for all workers, without loss of generality let it be the same for all $l_{j,t} = l_t$. The definition of L_t combined with the results above determines overtime as the solution of the equation:

$$L_t = A_t \left[\int_{1+z}^{\bar{A}} A_j d\Phi^E(A_j) + (l_t - z)\, N_t \right] - \kappa \tag{8}$$

Using the solutions for L_t, $q_{j,t}$ and $l_{j,t}$ in the objective function and rearranging gives the indirect utility function:

$$W(2) = \alpha \ln \alpha/b - \alpha + \alpha\gamma t + \alpha \ln(A_t) + b \int_{x_t}^{\bar{A}} A_j d\Phi(A_j) - b x_t N_t - b\kappa \tag{9}$$

where, recall: $x_t = 1 + z$, and $N_t = 1 - \Phi(x_t)$. The problems in stages 1 and 3 are almost identical, but the indirect utility function are instead:

$$W(1) = \alpha \ln \alpha/b - \alpha + \alpha\gamma t + \alpha \ln(A_t) + b \int_{x_t}^{\bar{A}} A_j d\Phi^N(A_j) - bx_t N_t. \tag{10}$$

$$W(3) = \alpha \ln \alpha/b - \alpha + \alpha\gamma t + \alpha \ln(A_t) + b \int_{x_t}^{\bar{A}} A_j d\Phi^E(A_j) - bx_t N_t - b\kappa A_\tau/A_t \tag{11}$$

Finally, the economy must choose whether to adopt a new technology now or in the next instant. Because we assumed that in the instant between the two, the probability that state 2 is reached is zero, the comparison involves comparing only $W(.)$ in state 3 with $W(.)$ in state 1. The technology will be adopted when:

$$W(1) \geq W(3) \Leftrightarrow A_t/A_\tau > A^*, \text{ with:} \tag{12}$$

$$A^* \equiv \kappa / \left[\int_{1+z}^{T} A_j \left(d\Phi^E(A_j) - d\Phi^N(A_j) \right) + (1+z)(\Phi^E(1+z) - \Phi^N(1+z)) \right] \tag{13}$$

This proves part c) of the proposition.

4.2 Proof of Proposition 2

At every instant, with a stock of employed workers N_t, the economy must decide on normal hours, overtime hours, and on the allocation of tasks. This static problem is identical to the one in proposition 1. Therefore, we still have $q_{j,t} = 1$ and $l_{j,t} = l_t$ for employed workers, and $L_t = \alpha A_t/b$ of total labor input. Combining this result with the production function proves part a).

To solve for employment and prove part b), we use a recursive representation. The indirect utility functions in (9)-(11) have several additive terms that do not affect the employment decisions. We can therefore drop them and write the problem using a system of Hamilton-Jacobi-Bellman equations:

$$\rho V(N,1) = \max_{F,H} \left[\int_{x_t}^{\bar{A}} A_j d\Phi^N(A_j) - (1+z)N - \beta F - H^{1/\nu} + E(dV)/dt \right], \tag{14}$$

$$\rho V(N,2) = \max_{F,H} \left[\int_{x_t}^{\bar{A}} A_j d\Phi^E(A_j) - (1+z)N - \kappa - \beta F - H^{1/\nu} + E(dV)/dt \right], \tag{15}$$

$$\rho V(N,3,t) = \max_{F,H} \left[\int_{x_t}^{\bar{A}} A_j d\Phi^E(A_j) - (1+z)N - \kappa e^{g_3 t} - \beta F - H^{1/\nu} + E(dV)/dt \right], \tag{16}$$

subject to the constraints: $H \geq 0$, $F \geq 0$, $N_t = 1 - \Phi(x_t)$ in states 2 and 3 and $N_t = 1 - \Gamma(x_t)$ in state 1. The value function $V(.)$ depends on employment N, the state of technology s, and for state 3 also on how long has it been since the start of the state t. The law of motion for N is $dN_t/dt = H_t - F_t - \chi N_t$, s jumps to 2 at rate μ_1 and to 3 at rate μ_2 and the arrival of stage 1 is chosen by the agent.

Using Ito's lemma, and letting $\mu_3 \equiv 0$:

$$E(dV)/dt = V_t(.)I(3) + V_N(.)\left(H - F - \chi N\right) + \mu_s\left(V(., s+1) - V(., s)\right), \tag{17}$$

where $I(s)$ is an indicator function equal to 1 in state s and 0 otherwise. Necessary conditions for optimality are:

$$-\beta - V_N(.) \leq 0,\ \ F \geq 0,\ \text{ w.a.l.o.e.} \tag{18}$$

$$-H^{1/\nu - 1}/\nu + V_N(.) \leq 0,\ \ H \geq 0,\ \text{ w.a.l.o.e.} \tag{19}$$

$$(\rho + \chi)V_N(., s) = x - 1 - z + V_{tN}I(3) + V_{NN}(., s)\left(H - F - \chi N\right) + \mu_s\left(V_N(., s+1) - V_N(., s)\right). \tag{20}$$

It is clearly never optimal to have positive hires and fires. If it were so, a policy that sets hires or fires to zero with the same net change in employment would produce the same outcome at lower cost. Thus, there are three optimal regions: (i) when there is firing, (ii) when there is neither firing nor hiring, and (iii) when there is hiring. Because $V(.)$ is concave, $V_N(.)$ must weakly fall with N so these three regions correspond to N being (i) above $\bar{N}^{(s)}$, (ii) between $\underline{N}^{(s)}$ and $\bar{N}^{(s)}$, and (iii) below $\underline{N}^{(s)}$. When there is positive hiring, there will be a steady-state level of employment $\hat{N}^{(s)}$ at which $H = \chi \hat{N}^{(s)}$. This proves part b).

Finally, denote by T the time stage 3 lasts. The optimal choice of when to engage in creative destruction and adopt a new technology is determined by value matching and smooth pasting conditions:

$$V(N_T, 3, T) = \tilde{V}(N_{t^*}, 1) \tag{21}$$

$$V_N(N_T, 3, T) = \tilde{V}_N(N_T, 1), \tag{22}$$

where $\tilde{V}(N, 1)$ only differs from $V(N, 1)$ in that we assumed that in the instant when there is a technology change, the probability of an immediate jump to stage 2 is zero. Using the Hamilton-Jacobi-Bellman equations, these conditions become:

$$\int_{x_s}^{\bar{A}} A_j d\Phi^E(A_j) - \kappa e^{g_3 s} = \int_{x_s}^{\bar{A}} A_j d\Phi^N(A_j), \tag{23}$$

Since $e^{g_3 T} = A_T / A_\tau$, then defining:

$$A^{**} = \kappa / \left[\int_{x_s}^{\bar{A}} A_j \left(d\Phi^E(A_j) - d\Phi^N(A_j) \right) \right] \tag{24}$$

proves proposition c).

4.3 Proof of Proposition 3

We split the choices of the representative agent in three steps. First, choose $q_{j,t}$, $l_{j,t}$ and L_t to maximize static period utility. Second, choose employment x_t and N_t to maximize intertemporal utility taking into account the adjustment costs. Third, choose consumption C_t to maximize intertemporal utility taking the budget constraint into account. At the end of the proposition, we verify that combining these sub-problems solves the full problem.

Step 1: The problem in the first step is similar to (1)-(3):

$$\max_{L_t, \{q_{j,t}, l_{j,t}\}} \left\{ \ln \left(\left(e^{\gamma_t} L_t \right)^\alpha K_t^{1-\alpha} - \delta K_t - \dot{K}_t \right) - b \int_x^{\bar{A}} \left(q_{j,t} + l_{j,t} \right) d\Phi^E(A_j) \right\} \tag{25}$$

$$s.t. \quad : \quad L_t = \int_{x_t}^T A_t (A_j q_{j,t} + l_{j,t} - z) d\Phi^E(A_j) - \kappa A_t - \beta F - H^{1/\nu} \tag{26}$$

$$0 \quad \leq \quad q_{j,t} \leq 1, \; l_{j,t} \geq 0. \tag{27}$$

The differences are in the objective function and the fact that x_t is not chosen. Working through the first-order conditions as before shows that $q_{j,t} = 1$, $l_{j,t} = l_t$ determined by (8), and $L_t = \alpha A_t Y_t / b C_t$. The indirect utility function is:

$$W(2) = \ln(C_t) - bL_t/A_t - b\beta F - bH^{1/\nu} + b \int_{x_t}^{\bar{A}} A_j d\Phi^E(A_j) - b(1+z)N_t - b\kappa.$$

The problem for stages 1 and 3 is similar with similar indirect utility functions.:

$$\begin{aligned} W(1) &= \ln(C_t) - bL_t/A_t - b\beta F - bH^{1/\nu} + b\int_{x_t}^{\bar{A}} A_j d\Phi^N(A_j) - b(1+z)N_t. \\ W(3) &= \ln(C_t) - bL_t/A_t - b\beta F - bH^{1/\nu} + b\int_{x_t}^{\bar{A}} A_j d\Phi^E(A_j) - b(1+z)N_t - b\kappa A_\tau/A_t \end{aligned} \tag{28}$$

Step 2: Using these indirect utility functions as objectives, note that the terms involving employment, hiring and firing, are the same as in the proof of Proposition 2. The solution is therefore the same, with the optimality conditions defining the three regions there. We now characterize the dynamics in each of these regions for each stage separately.

(Stage 1) If there is hiring, so we are below $\underline{N}^1$, (19) implies that $V_N(N,s) = (1/\nu)H^{1/\nu-1}$ and $H > 0$. Taking derivatives with respect to time of this expression yields:

$$dV_N/dt = V_{NN}\left(H - \chi N\right) = \left[(1-\nu)/\nu^2\right] H^{1/\nu-2} dH/dt. \tag{29}$$

Using this expression so substitute for V_{NN} in (20), we get:

$$\frac{dH/dt}{H} = \frac{\rho + \chi + \mu_1}{1/\nu - 1} - \frac{H^{1-1/\nu}\left(x - 1 - z + \mu_1 V_N(N,2)\right)}{\left(1/\nu - 1\right)/\nu} \tag{30}$$

Together with the link $N = 1 - \Phi^N(x)$ and the law of motion:

$$dN/dt = H - F - \chi N \tag{31}$$

this gives a system of two differential equations on a state variable (N) and a control variable (H). The steady state is $\hat{H} = \chi\hat{N}$ and

$$\hat{x}^{(1)} - 1 - z = (\rho + \chi + \mu_1)\left(\chi\hat{N}^{(1)}\right)^{1/\nu-1}/\nu - \mu_1 V_N(\hat{N}^{(1)}, 2) \tag{32}$$

The arm $dN/dt = 0$ is $H = \chi N$, an upward-sloping ray from the origin in (N, H) space. Below it, H is too small so N falls; above it, N rises. The arm $dH/dt = 0$ is

$$(\rho + \chi + \mu_1)\, H^{1/\nu-1}/\nu = x - 1 - z + \mu_1 V_N(N, 2). \tag{33}$$

Using the concavity of the value functions, it follows that this is downward-sloping. Above it, H is too large, so H rises; below it, H falls. Thus, along a saddle-path, H falls and N rises, so along the optimum trajectory $\partial H/\partial N < 0$. (Note that this confirms $V_{NN} < 0$.) The boundary conditions are the initial level of N_t and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t} V_N(N,1) = 0, \tag{34}$$

ensuring that the economy follows the saddle-path to the steady-state.

The system above applies while $H > 0$ and $V_N > 0$. When the saddle path hits the horizontal axis at employment level $\underline{N}^{(1)}$, hiring hits zero and $V_N = 0$. To the right of this point, the dynamics of employment follow $\dot{N} = -\chi N$ from its law of motion. Noting that $dV_N/dt = -\chi V_{NN} N$, condition (20) implies that in this region:

$$\dot{V}_N(N,1) = 1 + z - x + (\rho + \chi + \mu_1)V_N(N,1) - \mu_1 V_N(N,2), \tag{35}$$

as $V_N(N,1)$ falls monotonically with N from 0 to $-\beta$. It hits $-\beta$ at the employment level $\bar{N}^{(1)}$, where condition (18) states firing will start. If $N > \bar{N}^{(1)}$, there is firing $F = N - \bar{N}^{(1)}$, and employment jumps to $\bar{N}^{(1)}$.

(Stage 2) The analysis is exactly the same as in stage 1. The only differences are that now $N = 1 - \Phi^E(x)$ and $\mu_2 V_N(N,3,0)$ terms replace $\mu_1 V_N(N,2)$ terms.

(Stage 3) The dynamics are similar to those in stages 1 and 2. Using $N = 1 - \Phi^E(x)$ and deleting all terms involving μ, we get the same phase diagram and the same dynamics across regions of employment. However, the problem now has a finite horizon. The transversality condition is different and there is a further condition pinning down T. it is useful to write this problem in non-recursive form:

$$V(N_\tau,3,0) = \max_{T,\{N_t,F_t\}} \int_\tau^{\tau+T} e^{-\rho(t-\tau)} \left[\int_{x_t}^{\bar{A}} A_j d\Phi^E(A_j) - (1+z)N_t - \kappa e^{g_3 t} - \beta F_t - H_t^{1/\nu} \right] dt + e^{-\rho T}\tilde{V}(N_T,1) \tag{36}$$

$$s.t. \quad : \quad N_\tau = e^{\chi s} N_{\tau+t^*} + \int_\tau^{\tau+s} e^{\chi(t-\tau)} \left(H_t - F_t - \chi N_t\right) dt \tag{37}$$

The first-order condition with respect to T is.

$$\int_{x_T}^{\bar{A}} A_j d\Phi^E(A_j) - (1+z)N_T - \kappa e^{g_3 T} - \beta F_T - H_T^{1/\nu} + \lambda e^{\chi T}\left(H_T - F_T - \chi N_T\right) = \rho \tilde{V}(N_T,1) \tag{38}$$

where λ is the Lagrange multiplier. The first-order condition with respect to N_T is:

$$e^{-\rho T}\tilde{V}_N(N_T,1) = e^{\chi T}\lambda. \tag{39}$$

Replacing for λ, gives the value matching and smooth pasting conditions:

$$V(N_T,3,T) = \tilde{V}(N_T,1) \tag{40}$$

$$V_N(N_T,3,T) = \tilde{V}_N(N_T,1). \tag{41}$$

The value matching condition leads to the same technology adoption decision as in the proof of proposition 2:

$$Tg_3 = \ln\left(\int_{x_T}^{\bar{A}} A_j\left[d\Phi(A_j) - d\Gamma(A_j)\right]\right) - \ln(\kappa), \tag{42}$$

which links T to x_T (or N_T). In turn, the smooth pasting condition performs the role of the transversality condition from before.

Step 3: From the indirect utility function, ignoring additive terms that do not involve consumption, the problem is

$$\max_{C_t} \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(\ln C_t - bL_t/A_t\right)dt\right] \tag{43}$$

$$s.t. \quad : \quad dK_t/dt = \left(e^{\gamma_t}L_t\right)^\alpha K_t^{1-\alpha} - C_t - \delta K_t, \tag{44}$$

$$L_t = \alpha A_t Y_t / bC_t. \tag{45}$$

Defining $\hat{L}_t = A_t L_t$, it is easy to see the problem in part a) of the Proposition has identical first-order conditions to this one.

Final step: The optimal choices of normal hours and tasks to operate were independent from the other steps. The choice of optimal labor input depends on the choices at step 3, but the problem of step 3 takes this dependence into account. Finally, separating steps 2 and 3 is appropriate because in the indirect utility function F, H and C are additively separable and the law of motion for N_t is independent of C_t, while the law of motion for K_t is independent of N_t.

5 Algorithm to solve the model

Based on the proof of Proposition 3, we solve step 1 by evaluating the expressions, and step 3 by standard log-linearizations. Step 2 is more involved, and we use the following fixed-point algorithm:

Step 2.1) Assume a $V_N^{(0)}(N,1)$. We start with the constant $-\beta$.

Step 2.2) Start with stage 3 and work backwards. Fix an N_0. Guess a duration of stage 3, T, and solve for N_T using condition (42). Given the boundary conditions N_0 and N_T as well as T, solve the finite-time differential system in the proof of Proposition 3 for both employment and V_N. Check if $V_N(N_T,3,T) = V_N^{(0)}(N,1)$. If so, move on to step 2. Otherwise, lower/raise T until the equality holds.

Step 2.3) Store $V_N(N_0,3,0)$. Fix another N_0 and repeat step 1 until you have mapped the whole function $V_N^{(0)}(N,3,0)$ for values of N on a grid of 21 points on $[0.9,1]$.

Step 2.4) Move on to stage 2. Compute the saddle-path arm of the dynamic system and use it to map the path of N and $V_N^{(0)}(N,2)$.

Step 2.5) Move to stage 1. Compute the saddle-path arm of the dynamic system and use it to map the path of N and $V_N^{(1)}(N,1)$.

Step 2.6) Compute $\left\|V_N^{(1)} - V_N^{(0)}\right\|$ using a norm. We use the sum of squares. If is is below a small number (we use 10^{-6}) stop. Otherwise, return to step 2.

Having mapped the V_N as well as the time path for N_t starting from any N_0, we then take draws from the Poisson processes and simulate the path for employment.

6 Appendix

6.1 Turning point algorithms

Window method: For a given series, $\{x_t\}_{t=1}^T$, the window method with window size w begins by identifying dates in the range $[w+1, T-w]$ where $x_t \leq \min\left(\{x_s\}_{s=t-w}^{t+w}\right)$. Such dates are tentatively labeled as troughs. A similar operation yields a set of tentative peaks. The method then imposes the requirement that peaks and troughs alternate. This is achieved by retaining the latest of a series of successive turning points of the same type. We found that the window method was sensitive to noise and therefore pre-smoothed the data using a five-quarter, centered moving average.

Reversal method: The reversal method requires two parameters representing the "reversal pattern" that identifies a turning point. A $(3, 2)$ reversal (the one we use) identifies a peak as an episode in which the series rises for three successive quarters and then immediately falls for two successive quarters. Once a tentative set of turning points has been identified, the requirement that turning points alternate is imposed in the same manner as in the window method.

Bry-Boschan: The Bry-Boschan procedure is described by Bry and Boschan (1971) and King and Plosser (1994). It was originally developed for monthly data and we adapt it to quarterly data in the same manner as King and Plosser: the quarterly value is repeated for each month of the quarter. Our procedure differs from that described by King and Plosser in that we use a 10 month moving average in the first step and a 6 month moving average in the third. We use the programs made available by Monch and Uhlig (2004).

Chauvet-Hamilton: Chauvet and Hamilton (2005) fit a two-state Markov-switching model to the first differences of GDP in which each observation is drawn from normal distribution with a common variance and a mean that depends on the state. We expand their model to allow the variance of the first differences to change between states. As explained in the text, we also allow the variance to change before and after 1984Q3. Contractions are then defined as periods in which the smoothed regime probability is greater than 0.5 for the state with the smaller mean first difference. The remaining dates are classified as expansions. The model is estimated by numerical maximum likelihood.

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Table 1a. Duration of output and employment, linearly de-trended with breaks

			Average	t-statistic (p-value)	W-statistic (p-value)	Test equal asymmetry
Industrial Production	Window	Expansions	11.273	1.072	1.18	2.068*
		Contractions	8.167	(0.142)	(0.130)	(0.019)
	Reversal	Expansions	10.417	0.706	0.226	2.324**
		Contractions	8.250	(0.240)	(0.421)	(0.010)
	Bry-Boschan	Expansions	11.364	1.220	0.681	2.024*
		Contractions	7.546	(0.111)	(0.260)	(0.022)
	Regime-switching	Expansions	24.800	0.973	2.121*	-0.310
		Contractions	16.000	(0.165)	(0.041)	(0.622)
Employment Rate	Window	Expansions	17.875	2.401**	1.99*	
		Contractions	8.778	(0.008)	(0.037)	
	Reversal	Expansions	15.900	2.698**	2.357*	
		Contractions	6.600	(0.003)	(0.018)	
	Bry-Boschan	Expansions	18.375	2.754**	3.105**	
		Contractions	8.222	(0.003)	(0.006)	
	Regime-Switching	Expansions	15.222	1.505	1.208	
		Contractions	9.222	(0.066)	(0.129)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The test of equal asymmetry is for the null that the difference between expansions and contractions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 1b. Duration of output and employment, polynomially de-trended

			Average	t-statistic (p-value)	W-statistic (p-value)	Test equal asymmetry
Industrial Production	Window	Expansions	10.818	0.760	0.423	2.173*
		Contractions	8.583	(0.224)	(0.347)	(0.015)
	Reversal	Expansions	10.583	0.774	0.511	1.820*
		Contractions	8.167	(0.219)	(0.315)	(0.034)
	Bry-Boschan	Expansions	11.273	1.062	0.362	2.183*
		Contractions	8.000	(0.144)	(0.370)	(0.015)
	Regime-switching	Expansions	24.400	0.900	2.121*	-0.268
		Contractions	16.333	(0.184)	(0.041)	(0.606)
Employment Rate	Window	Expansions	17.625	2.318*	1.99*	
		Contractions	9.000	(0.010)	(0.037)	
	Reversal	Expansions	15.300	2.39**	2.041*	
		Contractions	7.200	(0.008)	(0.032)	
	Bry-Boschan	Expansions	18.000	2.633**	3.45**	
		Contractions	8.000	(0.004)	(0.003)	
	Regime-Switching	Expansions	15.111	1.435	1.016	
		Contractions	9.444	(0.076)	(0.170)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The test of equal asymmetry is for the null that the difference between expansions and contractions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 1c. Duration of output and employment, band-pass filter de-trended

			Average	t-statistic (p-value)	W-statistic (p-value)	Test equal asymmetry
Industrial Production	Window	Expansions	8.615	0.687	0.82	4.269**
		Contractions	7.714	(0.246)	(0.215)	(0.000)
	Reversal	Expansions	6.579	1.308	1.531	6.742**
		Contractions	5.263	(0.096)	(0.069)	(0.000)
	Bry-Boschan	Expansions	9.000	1.145	1.71	4.855**
		Contractions	7.357	(0.126)	(0.052)	(0.000)
	Regime-switching	Expansions	11.600	0.531	0.767	0.953
		Contractions	9.455	(0.298)	(0.234)	(0.170)
Employment Rate	Window	Expansions	15.000	2.085*	1.620	
		Contractions	8.500	(0.019)	(0.067)	
	Reversal	Expansions	15.300	2.390**	2.041*	
		Contractions	7.200	(0.008)	(0.032)	
	Bry-Boschan	Expansions	16.111	2.612**	2.773**	
		Contractions	7.500	(0.005)	(0.009)	
	Regime-Switching	Expansions	15.222	1.512	1.307	
		Contractions	9.222	(0.065)	(0.111)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The test of equal asymmetry is for the null that the difference between expansions and contractions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 1d. Duration of output and employment, modified-HP filter de-trended

			Average	t-statistic (p-value)	W-statistic (p-value)	Test equal asymmetry
Industrial Production	Window	Expansions	11.273	1.072	1.18	1.660*
		Contractions	8.167	(0.142)	(0.130)	(0.048)
	Reversal	Expansions	9.615	0.724	0.685	2.209*
		Contractions	7.615	(0.235)	(0.256)	(0.014)
	Bry-Boschan	Expansions	10.000	1.083	0.627	2.978**
		Contractions	7.333	(0.139)	(0.276)	(0.002)
	Regime-switching	Expansions	25.000	1.023	2.121*	-0.366
		Contractions	15.833	(0.153)	(0.041)	(0.643)
Employment Rate	Window	Expansions	17.250	2.17*	1.736	
		Contractions	9.333	(0.015)	(0.057)	
	Reversal	Expansions	15.300	2.39**	2.041*	
		Contractions	7.200	(0.008)	(0.032)	
	Bry-Boschan	Expansions	18.000	2.633**	3.45**	
		Contractions	8.000	(0.004)	(0.003)	
	Regime-Switching	Expansions	15.222	1.491	1.016	
		Contractions	9.333	(0.068)	(0.170)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The test of equal asymmetry is for the null that the difference between expansions and contractions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 1e. Duration of output and employment, alternative bandpass filter de-trended

			Average	t-statistic (p-value)	W-statistic (p-value)	Test equal asymmetry
Industrial Production BP (2,80)	Window	Expansions	10.364	0.544	0.731	1.901*
		Contractions	8.917	(0.293)	(0.243)	(0.029)
	Reversal	Expansions	9.846	0.878	1.054	2.104*
		Contractions	7.462	(0.190)	(0.155)	(0.018)
	Bry-Boschan	Expansions	10.455	0.628	0.484	2.546**
		Contractions	8.750	(0.265)	(0.325)	(0.005)
	Regime-switching	Expansions	13.571	0.293	0.224	1.150
		Contractions	15.625	(0.385)	(0.433)	(0.125)
Employment Rate BP (6,32)	Window	Expansions	9.917	1.93*	2.023*	1.067
		Contractions	7.615	(0.027)	(0.030)	(0.143)
	Reversal	Expansions	7.177	1.016	1.416	0.197
		Contractions	6.059	(0.155)	(0.085)	(0.422)
	Bry-Boschan	Expansions	10.417	2.616**	3.391**	1.236
		Contractions	7.000	(0.005)	(0.002)	(0.108)
	Regime-Switching	Expansions	13.111	1.025	1.243	0.288
		Contractions	9.800	(0.153)	(0.121)	(0.387)

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The test of equal asymmetry is for the null that the difference between expansions and contractions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 2a. Violence of output and employment, linearly de-trended with breaks

		Steepness			Sharpness			Slope		
		Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)
Industrial Production										
Window	Exp.	0.014	0.358	0.423	0.021	0.606	0.731	0.015	0.067	0
	Cont.	-0.015	(0.360)	(0.347)	0.024	(0.272)	(0.243)	-0.015	(0.473)	(0.500)
Reversal	Exp.	0.015	0.57	0.627	0.020	1.082	1.288	0.013	0.018	0.113
	Cont.	-0.017	(0.284)	(0.276)	0.025	(0.140)	(0.110)	-0.012	(0.493)	(0.466)
Bry-	Exp.	0.017	0.074	0.16	0.022	0.771	0.814	0.016	0.401	0.16
Boschan	Cont.	-0.018	(0.471)	(0.449)	0.026	(0.220)	(0.219)	-0.014	(0.344)	(0.449)
Regime-	Exp.	0.007	2.041*	2.121*	0.012	2.797**	4.118**	0.007	1.381	1.326
switching	Cont.	-0.016	(0.021)	(0.041)	0.029	(0.003)	(0.004)	-0.016	(0.084)	(0.123)
Employment Rate										
Window	Exp.	0.002	2.5**	2.416*	0.004	2.414**	2.573*	0.003	1.977*	2.126*
	Cont.	-0.004	(0.006)	(0.018)	0.006	(0.008)	(0.014)	-0.004	(0.024)	(0.030)
Reversal	Exp.	0.002	3.22**	3.717**	0.003	3.291**	3.717**	0.002	3.562**	3.896**
	Cont.	-0.005	(0.001)	(0.001)	0.006	(0.001)	(0.001)	-0.004	(0.000)	(0.001)
Bry-	Exp.	0.002	2.953**	3.53**	0.004	2.552**	2.416*	0.002	2.901**	3.309**
Boschan	Cont.	-0.005	(0.002)	(0.003)	0.006	(0.005)	(0.018)	-0.004	(0.002)	(0.004)
Regime-	Exp.	0.001	2.922**	2.075*	0.002	6.493**	6.971**	0.001	2.798**	2.2*
switching	Cont.	-0.002	(0.002)	(0.031)	0.006	(0.000)	(0.000)	-0.003	(0.003)	(0.025)
Output & employment										
Window	t-statistic		1.582			1.331			1.589	
	(p-value)		(0.057)			(0.092)			(0.056)	
Reversal	t-statistic		2.085*			1.484			2.47**	
	(p-value)		(0.019)			(0.069)			(0.007)	
Bry-	t-statistic		1.914*			1.313			1.951*	
Boschan	(p-value)		(0.028)			(0.095)			(0.026)	
Regime-	t-statistic		0.304			0.913			0.285	
switching	(p-value)		(0.381)			(0.181)			(0.388)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The last panel has the test of the null that the ratio of the violence in contractions and expansions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 2b. Violence of output and employment, polynomial de-trended

		Steepness			Sharpness			Slope		
		Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)
Industrial Production										
Window	Exp.	0.014	0.303	0.423	0.022	0.553	0.668	0.015	0.06	0.241
	Cont.	-0.015	(0.381)	(0.347)	0.024	(0.290)	(0.263)	-0.015	(0.476)	(0.416)
Reversal	Exp.	0.015	0.671	0.743	0.020	1.105	1.164	0.011	0.674	0.454
	Cont.	-0.018	(0.251)	(0.239)	0.026	(0.135)	(0.133)	-0.013	(0.250)	(0.335)
Bry-	Exp.	0.018	0.059	0.362	0.022	0.702	0.668	0.015	0.167	0.12
Boschan	Cont.	-0.018	(0.476)	(0.370)	0.025	(0.242)	(0.263)	-0.015	(0.434)	(0.464)
Regime-	Exp.	0.007	2.085*	1.825	0.013	2.703**	4.118**	0.008	1.431	1.562
switching	Cont.	-0.016	(0.019)	(0.063)	0.028	(0.003)	(0.004)	-0.016	(0.076)	(0.089)
Employment Rate										
Window	Exp.	0.002	2.536**	1.99*	0.004	2.441**	2.416*	0.002	2.081*	2.739*
	Cont.	-0.004	(0.006)	(0.037)	0.006	(0.007)	(0.018)	-0.004	(0.019)	(0.010)
Reversal	Exp.	0.002	3.033**	3.24**	0.003	3.201**	3.717**	0.002	3.325**	3.391**
	Cont.	-0.005	(0.001)	(0.003)	0.006	(0.001)	(0.001)	-0.004	(0.000)	(0.003)
Bry-	Exp.	0.002	3.281**	3.45**	0.003	2.881**	2.781*	0.002	3.055**	3.45**
Boschan	Cont.	-0.005	(0.001)	(0.004)	0.007	(0.002)	(0.010)	-0.004	(0.001)	(0.004)
Regime-	Exp.	0.001	2.839**	1.955*	0.002	6.341**	6.971**	0.001	2.616**	1.839*
switching	Cont.	-0.002	(0.002)	(0.039)	0.006	(0.000)	(0.000)	-0.003	(0.004)	(0.047)
Output & employment										
Window	t-statistic		1.878*			1.422			1.623	
	(p-value)		(0.030)			(0.078)			(0.052)	
Reversal	t-statistic		1.835*			1.380			2.076*	
	(p-value)		(0.033)			(0.084)			(0.019)	
Bry-	t-statistic		2.271*			1.608			2.176*	
Boschan	(p-value)		(0.012)			(0.054)			(0.015)	
Regime-	t-statistic		0.660			1.112			0.539	
switching	(p-value)		(0.255)			(0.133)			(0.295)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The last panel has the test of the null that the ratio of the violence in contractions and expansions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 2c. Violence of output and employment, band-pass filter de-trended

		Steepness			Sharpness			Slope		
		Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)
Industrial Production										
Window	Exp.	0.013	0.486	0.43	0.016	0.641	0.623	0.013	0.295	0.334
	Cont.	-0.015	(0.313)	(0.342)	0.018	(0.261)	(0.275)	-0.014	(0.384)	(0.378)
Reversal	Exp.	0.012	0.679	0.977	0.013	0.752	0.769	0.010	0.788	0.74
	Cont.	-0.014	(0.249)	(0.170)	0.016	(0.226)	(0.226)	-0.012	(0.215)	(0.235)
Bry-Boschan	Exp.	0.013	0.804	0.721	0.015	0.796	0.77	0.012	0.698	0.672
	Cont.	-0.016	(0.211)	(0.244)	0.019	(0.213)	(0.229)	-0.015	(0.243)	(0.259)
Regime-switching	Exp.	0.005	1.534	1.367	0.008	2.49**	2.543*	0.004	1.479	1.367
	Cont.	-0.010	(0.063)	(0.099)	0.017	(0.006)	(0.012)	-0.009	(0.070)	(0.099)
Employment Rate										
Window	Exp.	0.002	1.487	1.334	0.004	1.936*	1.62	0.003	1.356	1.243
	Cont.	-0.003	(0.069)	(0.106)	0.006	(0.026)	(0.067)	-0.004	(0.088)	(0.121)
Reversal	Exp.	0.002	2.849**	3.097**	0.003	3.055**	3.097**	0.002	3.116**	2.830**
	Cont.	-0.005	(0.002)	(0.005)	0.006	(0.001)	(0.005)	-0.004	(0.001)	(0.007)
Bry-Boschan	Exp.	0.002	2.383**	2.147*	0.004	2.199*	1.72	0.002	1.795*	1.720
	Cont.	-0.004	(0.009)	(0.027)	0.006	(0.014)	(0.056)	-0.004	(0.036)	(0.056)
Regime-switching	Exp.	0.001	2.624**	2.608*	0.002	6.096**	6.971**	0.001	2.762**	2.200*
	Cont.	-0.002	(0.004)	(0.012)	0.006	(0.000)	(0.000)	-0.003	(0.003)	(0.025)
Output & employment										
Window	t-statistic		0.828			0.933			0.815	
	(p-value)		(0.204)			(0.175)			(0.208)	
Reversal	t-statistic		1.719*			1.426			1.515	
	(p-value)		(0.043)			(0.077)			(0.065)	
Bry-Boschan	t-statistic		1.128			0.987			0.736	
	(p-value)		(0.130)			(0.162)			(0.231)	
Regime-switching	t-statistic		0.474			0.876			0.21	
	(p-value)		(0.318)			(0.191)			(0.417)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The last panel has the test of the null that the ratio of the violence in contractions and expansions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 2d. Violence of output and employment, modified-HP filter de-trended

		Steepness			Sharpness			Slope		
		Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)
Industrial Production										
Window	Exp.	0.014	0.206	0.241	0.022	0.529	0.668	0.016	0.201	0.181
	Cont.	-0.015	(0.418)	(0.416)	0.024	(0.299)	(0.263)	-0.015	(0.420)	(0.440)
Reversal	Exp.	0.015	0.35	0.378	0.020	0.772	0.947	0.013	0.238	0.378
	Cont.	-0.016	(0.363)	(0.362)	0.023	(0.220)	(0.181)	-0.012	(0.406)	(0.362)
Bry-	Exp.	0.017	0.071	0.057	0.021	0.587	0.569	0.015	0.522	0.17
Boschan	Cont.	-0.016	(0.472)	(0.489)	0.024	(0.279)	(0.295)	-0.013	(0.301)	(0.444)
Regime-	Exp.	0.007	2.148*	2.121*	0.012	2.852**	4.118**	0.007	1.394	1.326
switching	Cont.	-0.017	(0.016)	(0.041)	0.029	(0.002)	(0.004)	-0.017	(0.082)	(0.123)
Employment Rate										
Window	Exp.	0.002	2.291*	2.267*	0.004	2.314*	2.126*	0.003	1.847*	2.126*
	Cont.	-0.004	(0.011)	(0.023)	0.006	(0.010)	(0.030)	-0.004	(0.032)	(0.030)
Reversal	Exp.	0.002	2.995**	3.391**	0.003	3.176**	3.549**	0.002	3.266**	3.24**
	Cont.	-0.005	(0.001)	(0.003)	0.006	(0.001)	(0.002)	-0.004	(0.001)	(0.003)
Bry-	Exp.	0.002	3.222**	3.207**	0.004	2.859**	2.592*	0.002	3.001**	3.45**
Boschan	Cont.	-0.005	(0.001)	(0.005)	0.006	(0.002)	(0.014)	-0.004	(0.001)	(0.004)
Regime-	Exp.	0.001	3.118**	2.757**	0.002	6.394**	6.971**	0.001	2.914**	2.200*
switching	Cont.	-0.002	(0.001)	(0.009)	0.007	(0.000)	(0.000)	-0.003	(0.002)	(0.025)
Output & employment										
Window	t-statistic		1.537			1.298			1.569	
	(p-value)		(0.062)			(0.097)			(0.058)	
Reversal	t-statistic		1.981*			1.458			2.491**	
	(p-value)		(0.024)			(0.072)			(0.006)	
Bry-	t-statistic		1.925*			1.381			1.922*	
Boschan	(p-value)		(0.027)			(0.084)			(0.027)	
Regime-	t-statistic		0.149			0.878			0.191	
switching	(p-value)		(0.441)			(0.190)			(0.424)	

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The last panel has the test of the null that the ratio of the violence in contractions and expansions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 2e. Violence of output and employment, alternative band-pass filter de-trended

		Steepness			Sharpness			Slope		
		Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)	Average	t-statistic (p-value)	W-statistic (p-value)
Industrial Production BP (2,80)										
Window	Exp.	0.014	0.027	0.00	0.021	0.515	0.793	0.016	0.364	0.301
	Cont.	-0.014	(0.489)	(0.500)	0.024	(0.303)	(0.225)	-0.015	(0.358)	(0.393)
Reversal	Exp.	0.015	0.324	0.378	0.020	0.737	0.789	0.013	0.313	0.327
	Cont.	-0.016	(0.373)	(0.362)	0.023	(0.231)	(0.224)	-0.012	(0.377)	(0.381)
Bry-Boschan	Exp.	0.018	0.232	0.06	0.022	0.492	0.668	0.017	0.634	0.241
	Cont.	-0.017	(0.408)	(0.488)	0.025	(0.312)	(0.263)	-0.014	(0.263)	(0.416)
Regime-switching	Exp.	0.010	0.19	0.45	0.013	1.397	0.921	0.010	0.044	0.224
	Cont.	-0.011	(0.425)	(0.347)	0.021	(0.081)	(0.198)	-0.011	(0.483)	(0.433)
Employment Rate BP (6,32)										
Window	Exp.	0.002	0.764	0.866	0.003	0.906	1.035	0.002	0.672	0.755
	Cont.	-0.003	(0.222)	(0.203)	0.004	(0.183)	(0.160)	-0.003	(0.251)	(0.235)
Reversal	Exp.	0.003	0.625	0.425	0.003	0.744	0.528	0.002	0.736	0.528
	Cont.	-0.003	(0.266)	(0.342)	0.003	(0.228)	(0.305)	-0.003	(0.231)	(0.305)
Bry-Boschan	Exp.	0.002	1.276	1.15	0.003	1.147	0.978	0.002	1.239	1.035
	Cont.	-0.003	(0.101)	(0.135)	0.004	(0.126)	(0.174)	-0.003	(0.108)	(0.160)
Regime-switching	Exp.	0.001	1.341	0.979	0.002	3.209**	2.773**	0.001	1.832*	1.72
	Cont.	-0.002	(0.090)	(0.178)	0.004	(0.001)	(0.009)	-0.002	(0.034)	(0.056)
Output & employment			ER(6,32)	IP (2,80)		ER(6,32)	IP (2,80)		ER(6,32)	IP (2,80)
Window	t-statistic		0.257	1.253		0.301	1.074		0.336	1.424
	(p-value)		(0.399)	(0.105)		(0.382)	(0.142)		(0.369)	(0.077)
Reversal	t-statistic		0.005	1.883*		0.039	1.466		0.025	2.396**
	(p-value)		(0.498)	(0.030)		(0.485)	(0.071)		(0.490)	(0.008)
Bry-Boschan	t-statistic		0.507	1.848*		0.412	1.28		0.491	1.815*
	(p-value)		(0.306)	(0.032)		(0.340)	(0.100)		(0.312)	(0.035)
Regime-switching	t-statistic		0.251	1.462		0.184	1.337		0.149	1.448
	(p-value)		(0.401)	(0.072)		(0.427)	(0.091)		(0.441)	(0.074)

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. The last panel has the test of the null that the ratio of the violence in contractions and expansions for output is the same as for employment. * and ** denote significance at the 5% and 1% levels respectively.

Table 3. Duration and violence of output and employment, with monthly observations

		Duration	Violence (Steepness)	Violence (Sharpness)	Violence (Slope)
Industrial Production	Average difference	-10.60	-0.009	0.003	-0.011
	Fraction of rejections	3/16	4/16	4/16	3/16
	at 5% level	3/16	4/16	4/16	4/16
	Fraction of rejections	3/16	3/16	4/16	3/16
	at 1% level	3/16	4/16	4/16	4/16
Employment Rate	Average difference	-23.15	-0.002	0.001	-0.002
	Fraction of rejections	11/16	11/16	6/16	10/16
	at 5% level	12/16	10/16	12/16	10/16
	Fraction of rejections	3/16	4/16	1/16	8/16
	at 1% level	8/16	7/16	4/16	10/16

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-tending and detecting turning points. Differences are contractions less expansions.

Table 4. Duration and violence of output, using different series

		Duration	Violence (Steepness)	Violence (Sharpness)	Violence (Slope)
GDP	Average difference	-5.30 (-1.64)	-0.013 (-0.012)	0.002 (0.001)	-0.011 (-0.010)
	Fraction of rejections at 5% level	5/16 (1/12) 4/16 (0/12)	4/16 (0/12) 4/16 (0/12)	3/16 (0/12) 4/16 (0/12)	4/16 (0/12) 4/16 (0/12)
	Fraction of rejections at 1% level	1/16 (0/12) 4/16 (0/12)	4/16 (0/12) 4/16 (0/12)	1/16 (0/12) 2/16 (0/12)	4/16 (0/12) 4/16 (0/12)
Non-farm Business Output	Average difference	-6.02 (-1.17)	-0.016 (-0.017)	0.000 (0.000)	-0.014 (-0.015)
	Fraction of rejections at 5% level	4/16 (0/12) 4/16 (0/12)	4/16 (0/12) 3/16 (0/12)	0/16 (0/12) 0/16 (0/12)	3/16 (0/12) 3/16 (0/12)
	Fraction of rejections at 1% level	3/16 (0/12) 3/16 (0/12)	3/16 (0/12) 3/16 (0/12)	0/16 (0/12) 0/16 (0/12)	3/16 (0/12) 3/16 (0/12)
Real Sales	Average difference	-6.12 (-0.90)	-0.017 (-0.019)	0.006 (0.002)	-0.011 (-0.014)
	Fraction of rejections at 5% level	2/16 (0/12) 2/16 (0/12)	1/16 (0/12) 3/16 (0/12)	3/16 (1/12) 3/16 (0/12)	0/16 (0/12) 1/16 (0/12)
	Fraction of rejections at 1% level	1/16 (0/12) 2/16 (0/12)	1/16 (0/12) 2/16 (0/12)	2/16 (0/12) 2/16 (0/12)	0/16 (0/12) 0/16 (0/12)
Real Personal Income	Average difference	-3.53 (-1.98)	-0.009 (-0.009)	0.002 (0.001)	-0.008 (-0.008)
	Fraction of rejections at 5% level	3/16 (0/12) 3/16 (0/12)	3/16 (0/12) 3/16 (0/12)	3/16 (0/12) 4/16 (1/12)	3/16 (0/12) 3/16 (0/12)
	Fraction of rejections at 1% level	0/16 (0/12) 0/16 (0/12)	3/16 (0/12) 3/16 (0/12)	3/16 (0/12) 3/16 (0/12)	3/16 (0/12) 3/16 (0/12)
Consumption	Average difference	-16.20 (-1.45)	-0.009 (-0.010)	0.003 (0.001)	-0.007 (-0.008)
	Fraction of rejections at 5% level	1/16 (0/12) 0/16 (0/12)	0/16 (0/12) 3/16 (0/12)	0/16 (0/12) 0/16 (0/12)	0/16 (0/12) 1/16 (0/12)
	Fraction of rejections at 1% level	0/16 (0/12) 0/16 (0/12)	0/16 (0/12) 3/16 (0/12)	0/16 (0/12) 0/16 (0/12)	0/16 (0/12) 1/16 (0/12)
Investment	Average difference	-1.19 (-0.65)	-0.052 (-0.064)	0.005 (0.002)	-0.043 (-0.051)
	Fraction of rejections at 5% level	0/16 (0/12) 1/16 (1/12)	0/16 (0/12) 0/16 (0/12)	4/16 (0/12) 4/16 (0/12)	0/16 (0/12) 0/16 (0/12)
	Fraction of rejections at 1% level	0/16 (0/12) 0/16 (0/12)	0/16 (0/12) 0/16 (0/12)	3/16 (0/12) 4/16 (0/12)	0/16 (0/12) 0/16 (0/12)
Government Spending	Average difference	21.03 (1.64)	-0.013 (-0.014)	-0.005 (-0.002)	-0.013 (-0.013)
	Fraction of rejections at 5% level	0/16 (0/12) 3/16 (0/12)	1/16 (0/12) 5/16 (1/12)	0/16 (0/12) 0/16 (0/12)	3/16 (2/12) 3/16 (1/12)
	Fraction of rejections at 1% level	0/16 (0/12) 3/16 (0/12)	1/16 (0/12) 2/16 (0/12)	0/16 (0/12) 0/16 (0/12)	1/16 (0/12) 1/16 (0/12)

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-tending and detecting turning points. Differences are contractions less expansions. In parentheses are the results excluding the use of the Chauvet-Hamilton algorithm.

Table 5. Duration and violence of pre-war pig iron production

		Duration	Violence (Steepness)	Violence (Sharpness)	Violence (Slope)
Pig Iron Production 1877-1929	Average difference	-0.20	-0.10	0.018	-0.094
	Fraction of rejections at 5% level	10/16 1/16	2/16 4/16	3/16 3/16	1/16 2/16
	Fraction of rejections at 1% level	7/16 1/16	2/16 1/16	3/16 3/16	1/16 0/16

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-tending and detecting turning points. Differences are contractions less expansions.

Table 6. Duration and violence of employment, using different series

		Duration	Violence (Steepness)	Violence (Sharpness)	Violence (Slope)
Total Employment	Average difference	-5.35 (-6.88)	-0.007 (-0.008)	0.002 (0.002)	-0.006 (-0.007)
	Fraction of rejections	5/16 (6/16)	3/16 (4/16)	5/16 (6/16)	5/16 (5/16)
	at 5% level	4/16 (6/16)	4/16 (4/16)	7/16 (8/16)	4/16 (4/16)
	Fraction of rejections	3/16 (4/16)	3/16 (4/16)	4/16 (4/16)	3/16 (4/16)
	at 1% level	3/16 (4/16)	3/16 (4/16)	4/16 (4/16)	3/16 (4/16)
Total Employment (Payroll)	Average difference	-4.66 (-5.73)	-0.009 (-0.009)	0.003 (0.003)	-0.008 (-0.009)
	Fraction of rejections	5/16 (6/16)	5/16 (7/16)	8/16 (8/16)	4/16 (5/16)
	at 5% level	6/16 (8/16)	7/16 (10/16)	10/16 (12/16)	5/16 (7/16)
	Fraction of rejections	0/16 (0/16)	1/16 (1/16)	1/16 (2/16)	0/16 (0/16)
	at 1% level	0/16 (0/16)	1/16 (1/16)	3/16 (3/16)	0/16 (0/16)
Employment Rate 16 – 24 Yrs	Average difference	-4.32 (-4.82)	-0.010 (-0.010)	0.003 (0.004)	-0.009 (-0.009)
	Fraction of rejections	9/16 (10/16)	7/16 (7/16)	5/16 (5/16)	5/16 (4/16)
	at 5% level	9/16 (10/16)	5/16 (6/16)	6/16 (6/16)	4/16 (4/16)
	Fraction of rejections	3/16 (3/16)	3/16 (4/16)	4/16 (4/16)	3/16 (4/16)
	at 1% level	5/16 (5/16)	4/16 (5/16)	4/16 (4/16)	3/16 (4/16)
Employment Rate Over 25 Yrs	Average difference	-5.49 (-6.15)	-0.005 (-0.005)	0.002 (0.002)	-0.004 (-0.005)
	Fraction of rejections	8/16 (7/16)	7/16 (8/16)	13/16 (14/16)	8/16 (9/16)
	at 5% level	11/16 (11/16)	9/16 (11/16)	12/16 (15/16)	9/16 (11/16)
	Fraction of rejections	2/16 (1/16)	1/16 (1/16)	4/16 (4/16)	4/16 (4/16)
	at 1% level	5/16 (5/16)	8/16 (8/16)	7/16 (7/16)	6/16 (7/16)
Participation Rate	Average difference	2.51 (2.28)	-0.002 (-0.002)	0.000 (0.000)	-0.002 (-0.002)
	Fraction of rejections	3/16 (2/16)	2/16 (1/16)	1/16 (1/16)	2/16 (1/16)
	at 5% level	4/16 (3/16)	3/16 (2/16)	3/16 (3/16)	2/16 (2/16)
	Fraction of rejections	1/16 (0/16)	1/16 (0/16)	1/16 (0/16)	1/16 (0/16)
	at 1% level	1/16 (0/16)	2/16 (1/16)	1/16 (1/16)	1/16 (0/16)
Hours per Worker	Average difference	-2.24 (-3.18)	-0.005 (-0.005)	0.000 (0.000)	-0.004 (-0.003)
	Fraction of rejections	3/16 (4/16)	3/16 (4/16)	3/16 (4/16)	3/16 (4/16)
	at 5% level	2/16 (3/16)	5/16 (7/16)	3/16 (4/16)	4/16 (6/16)
	Fraction of rejections	2/16 (3/16)	3/16 (4/16)	1/16 (1/16)	2/16 (3/16)
	at 1% level	2/16 (3/16)	3/16 (4/16)	2/16 (3/16)	3/16 (4/16)

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-tending and detecting turning points. Differences are contractions less expansions. In parentheses are the results using the parameters (2,80) for the band-pass filter.

Table 7. Maximum likelihood estimates and tests on a statistical model

Panel A. Industrial Production		
Maximum-likelihood estimates	Estimates	Standard Errors
p_1	0.9441	0.1147
p_2	0.9198	0.1202
μ_1	0.0053	0.0010
μ_2	-0.0102	0.0024
$\sigma_{1,pre}$	0.0124	0.0015
$\sigma_{2,pre}$	0.0327	0.0029
$\sigma_{1,post}$	0.0066	0.0007
$\sigma_{2,post}$	0.0100	0.0014
Likelihood ratio tests	Statistics	p-values
$p_1=p_2$	0.46	0.50
$\mu_1= -\mu_2$	2.71	0.10
$\mu_1= -\mu_2$, $\sigma_{1,pre}=\sigma_{2,pre}$, $\sigma_{1,post}=\sigma_{2,post}$	46.40**	0.00
Panel B. Employment Rate		
Maximum-likelihood estimates	Estimates	Standard Errors
p_1	0.9237	0.1056
p_2	0.8869	0.1234
μ_1	0.0009	0.0002
μ_2	-0.0022	0.0005
$\sigma_{1,pre}$	0.0019	0.0002
$\sigma_{2,pre}$	0.0070	0.0007
$\sigma_{1,post}$	0.0014	0.0001
$\sigma_{2,post}$	0.0022	0.0003
Likelihood ratio tests	Statistics	p-values
$p_1=p_2$	0.77	0.38
$\mu_1= -\mu_2$	5.66*	0.02
$\mu_1= \mu_2$, $\sigma_{1,pre}=\sigma_{2,pre}$, $\sigma_{1,post}=\sigma_{2,post}$	71.36**	0.00

Notes: The likelihood function was maximised using a quasi-Newton method. * and ** denote significance at the 5% and 1% levels respectively.

Figure 1: CDF's for the duration of expansions and contractions in the employment rate

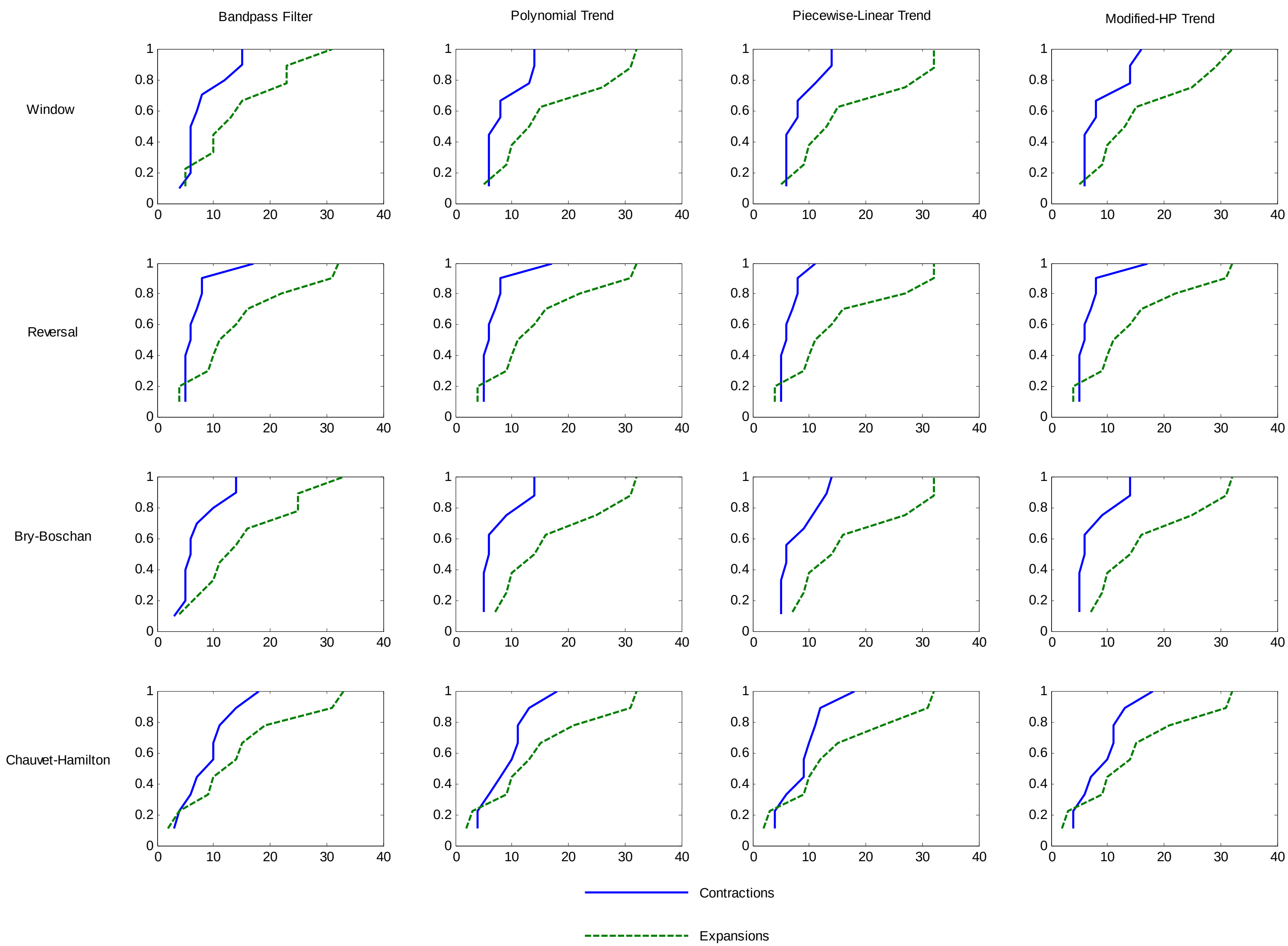

Figure 2: CDF's for the duration of expansions and contractions in industrial production

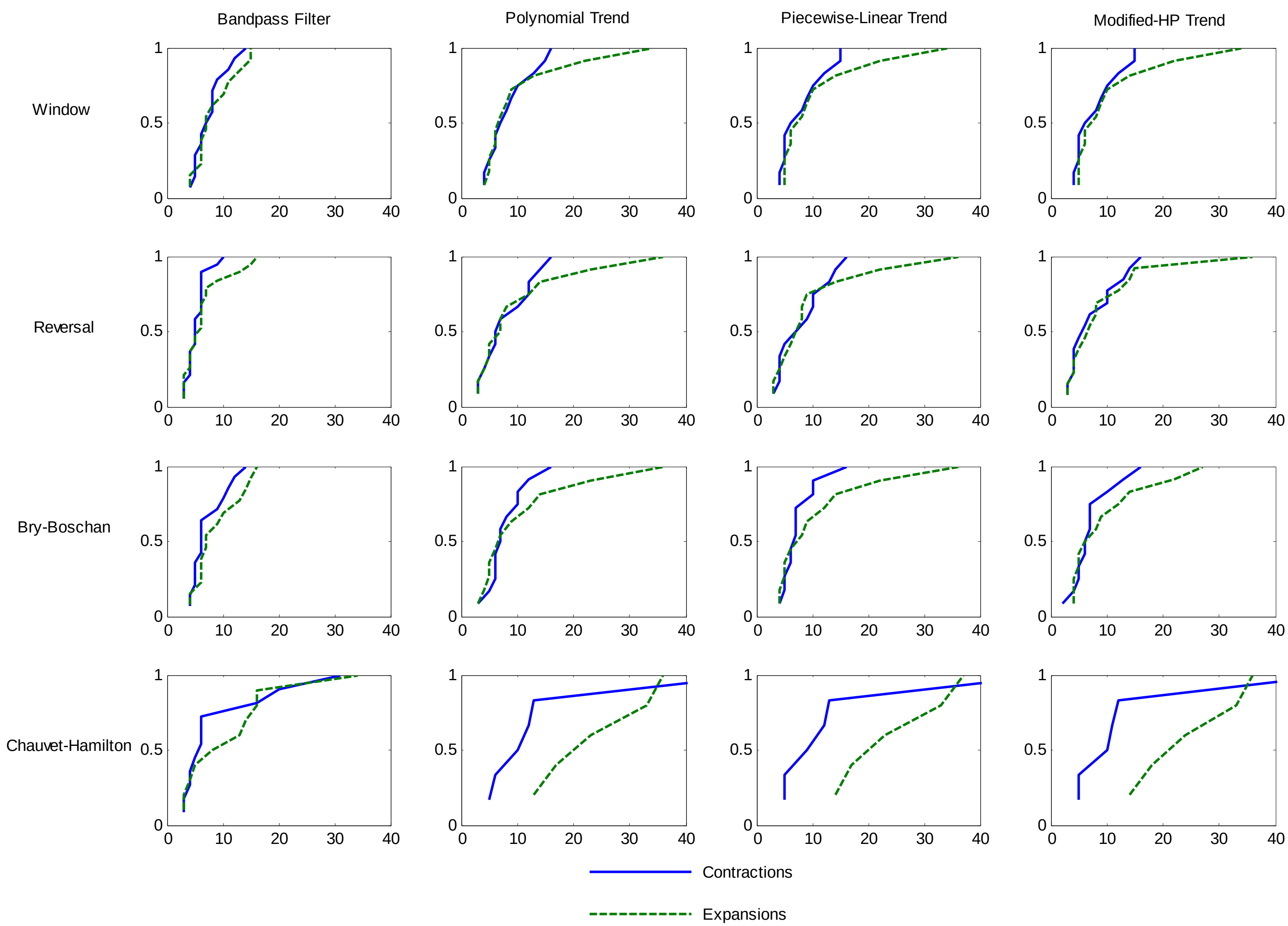

Figure 3a: CDF's for the steepness of expansions and contractions in the employment rate

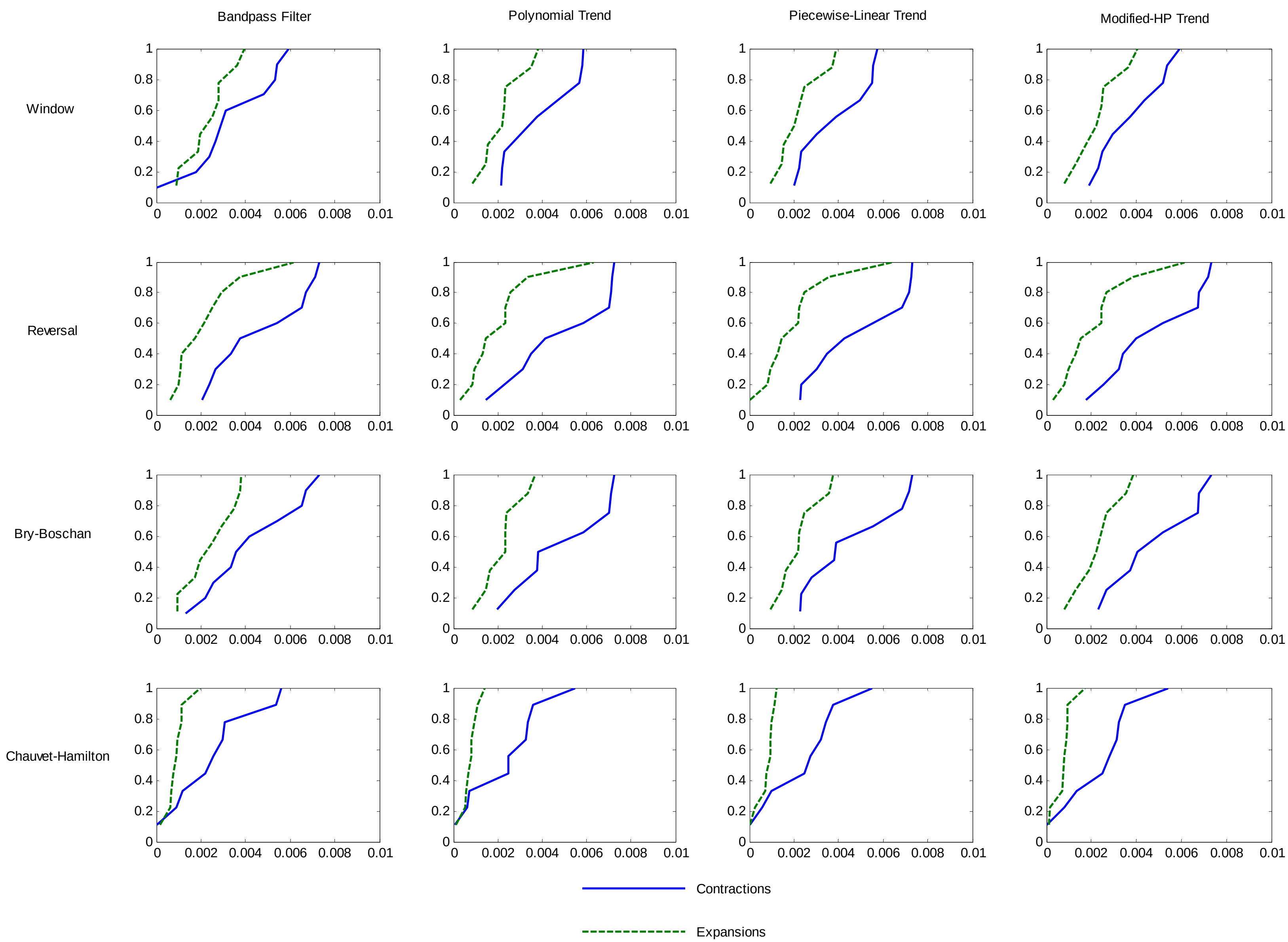

Figure 3b: CDF's for the sharpness of expansions and contractions in the employment rate

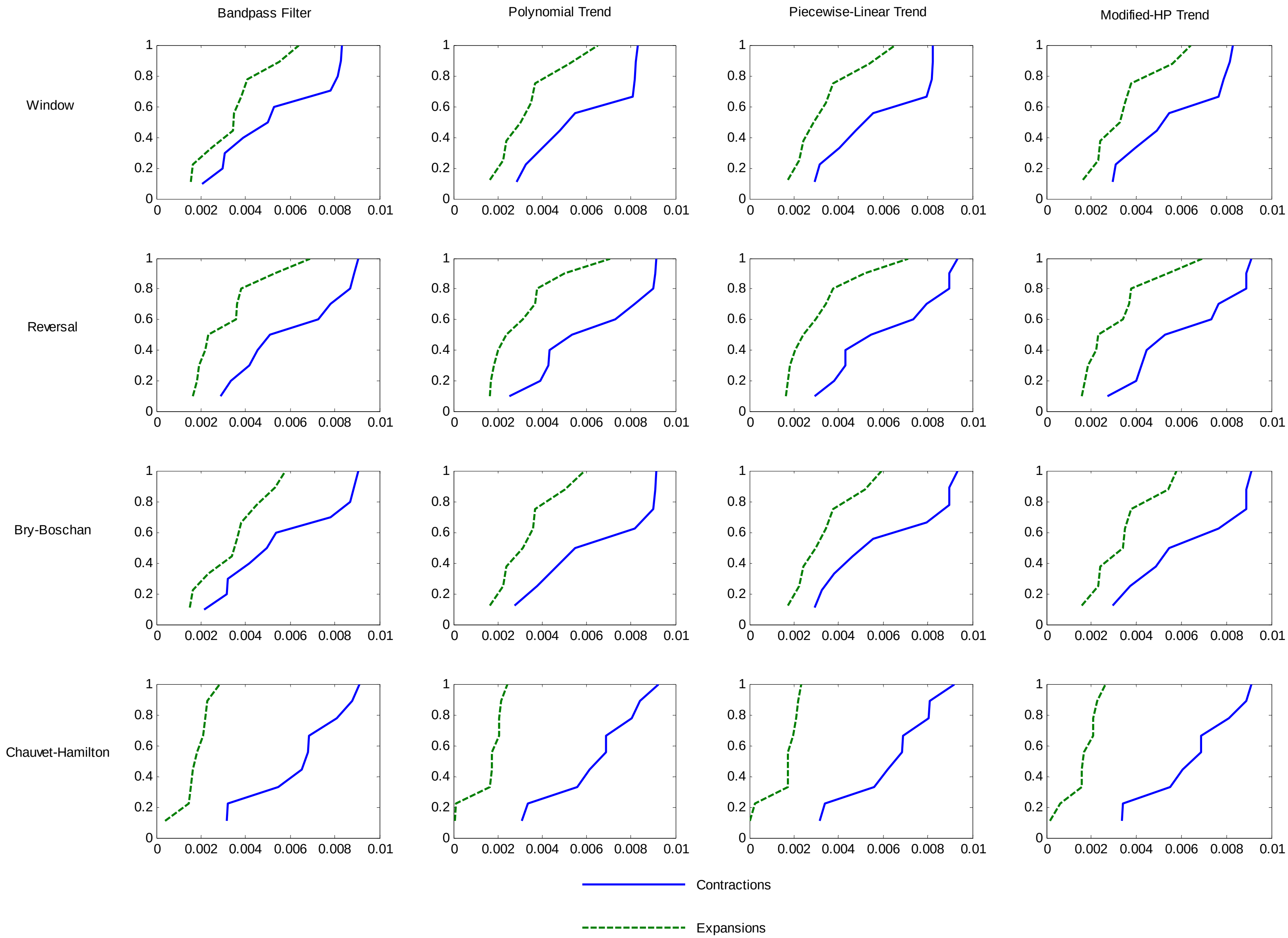

Figure 3c: CDF's for the slope of expansions and contractions in the employment rate

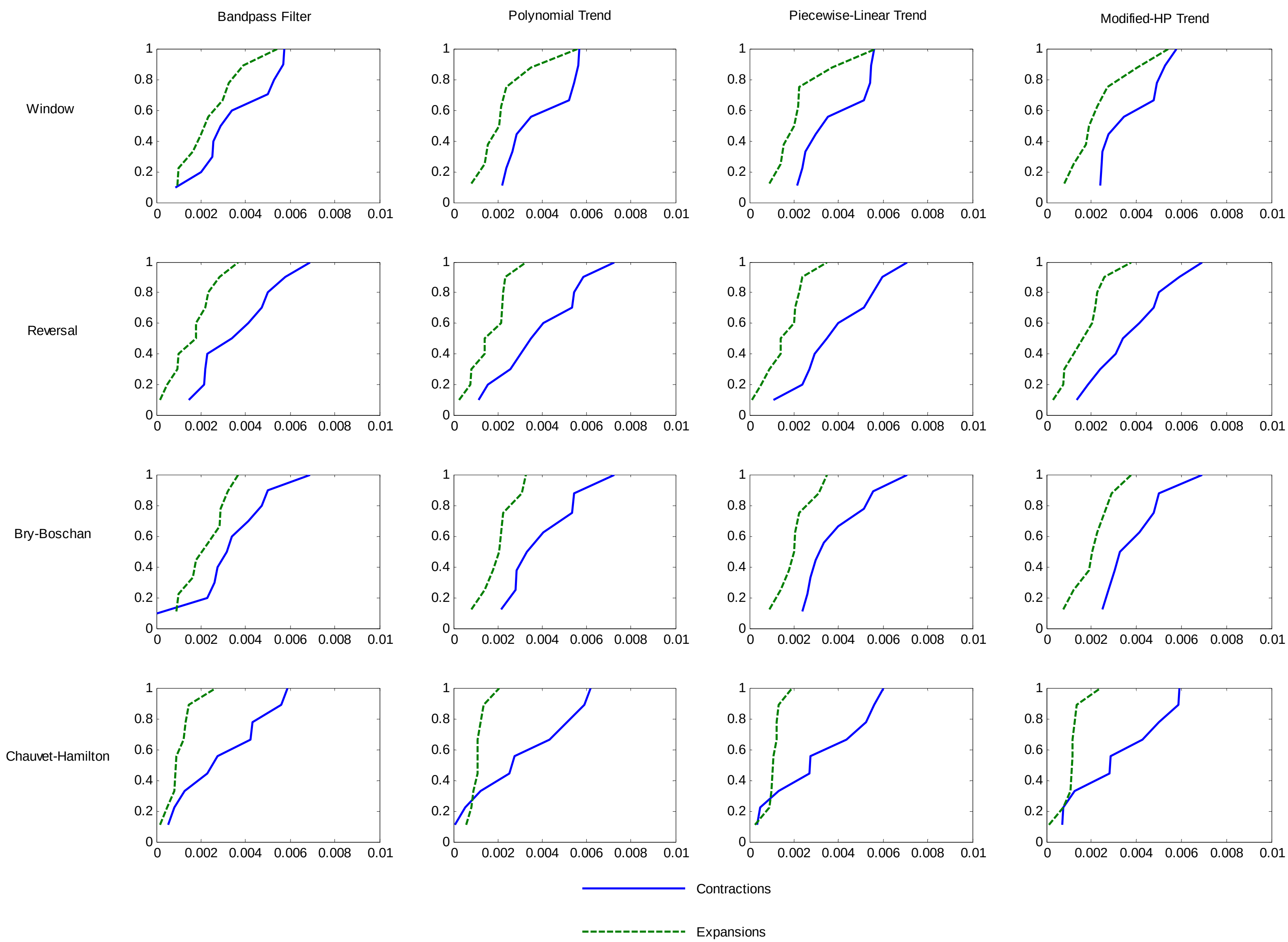

Figure 4a: CDF's for the steepness of expansions and contractions in industrial production

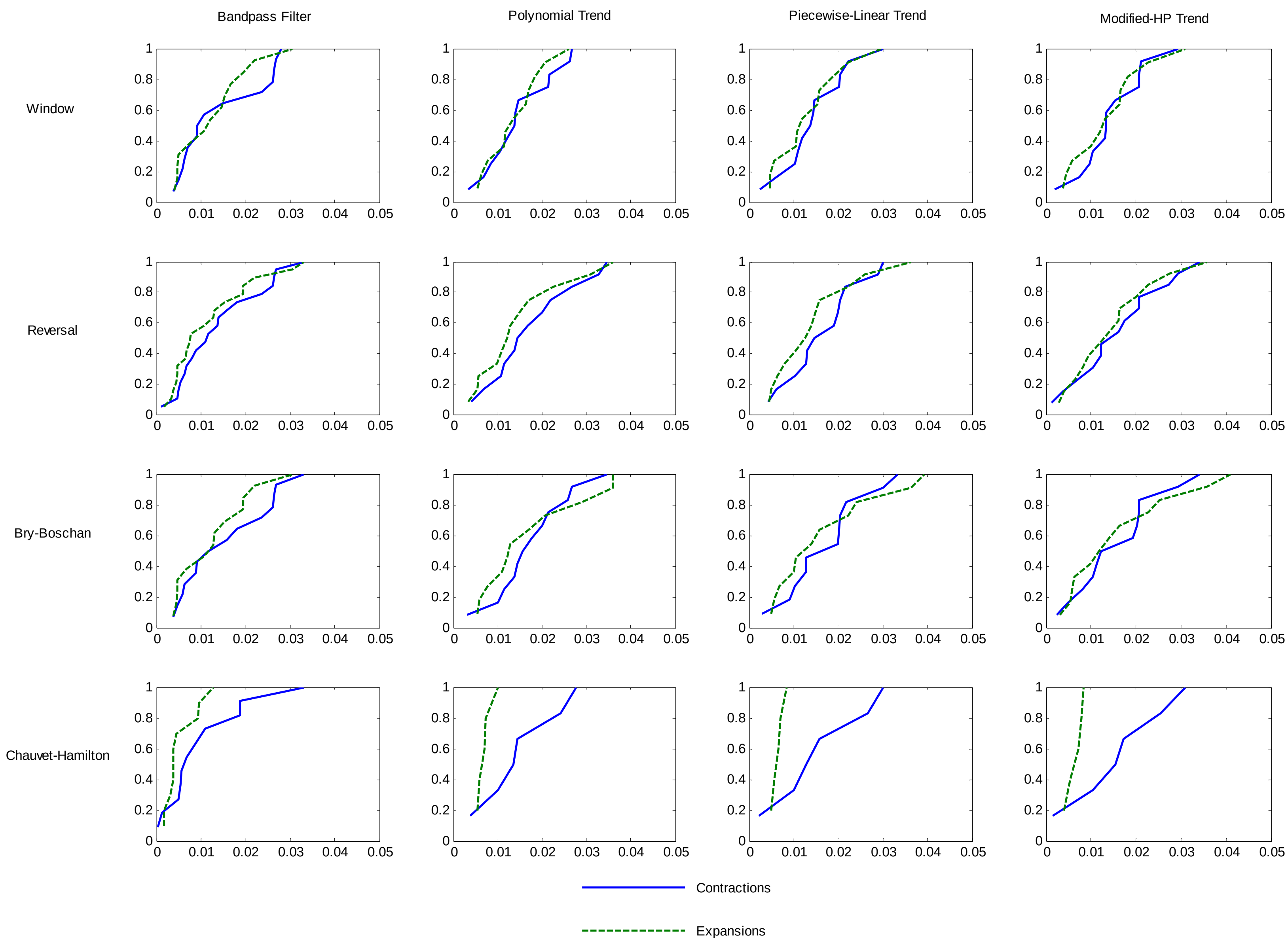

Figure 4b: CDF's for the sharpness of expansions and contractions in industrial production

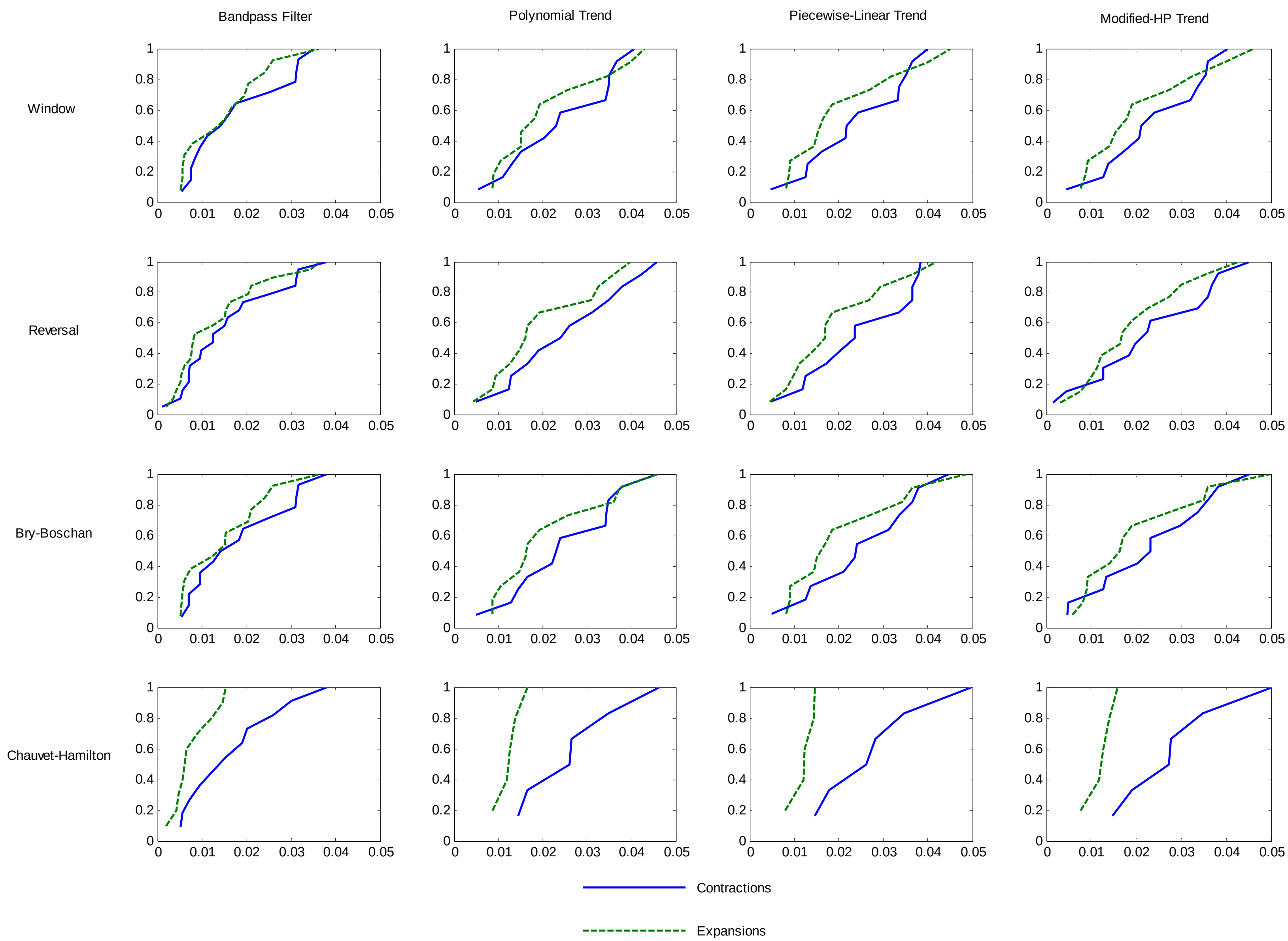

Figure 4c: CDF’s for the slope of expansions and contractions in industrial production