Time-Varying Idiosyncratic Risk and Aggregate Consumption Dynamics

Alisdair McKay*

Boston University

April 2017

Abstract

Long-term earnings losses for displaced workers are large and counter-cyclical. Similarly, the skewness of earnings growth rates is strongly pro-cyclical. This paper presents an incomplete markets business cycle model in which idiosyncratic risk varies over time in accordance with these empirical findings. These dynamics of idiosyncratic risk give rise to a cyclical precautionary savings motive that substantially raises the volatility of aggregate consumption growth. According to the model, idiosyncratic risk spiked during the Great Recession, leading to a substantial decline in aggregate consumption.

Keywords: consumption, idiosyncratic risk, incomplete markets, business cycle.

*Contact: amckay@bu.edu. I would like to thank Susanto Basu, Simon Gilchrist, Emi Nakamura, Ricardo Reis, Jón Steinsson, the editor and two anonymous referees, and seminar participants at Columbia, UCL, the Philadelphia Workshop on Macroeconomics, Federal Reserve Bank of St. Louis, NYU, the Green Line Macro meeting, and the Federal Reserve Bank of Minneapolis for helpful comments and suggestions.

1 Introduction

Recent empirical studies using large panel datasets on individual earnings portray recessions as times when individuals face substantially larger downside risks to their earnings prospects. Moreover, these risks appear to have highly persistent effects on an individual's earnings. Davis and von Wachter (2011) show that earnings losses from job-displacement are large, long-lasting, and roughly twice as large when the displacement occurs in a recession as opposed to an expansion. The differential impact of displacement in a recession is evident even twenty years after the event occurred. Similarly, Guvenen et al. (2014) show that the distribution of five-year earnings growth rates displays considerable pro-cyclical skewness, meaning severe negative events are more likely in a recession. According to this empirical evidence, recessions are times when workers face considerably more risk to their long-term earnings prospects.

The purpose of this paper is to explore how the cyclical dynamics of these risks alter the precautionary savings motive and the dynamics of aggregate consumption over the business cycle. The analysis uses a general equilibrium business cycle model with uninsurable idiosyncratic shocks to earnings. There are several sources of time-variation in idiosyncratic risks in the model. First, job-finding and separation rates vary over time, leading to fluctuations in the incidence and duration of unemployment spells. Second, workers who are laid off draw from a negatively skewed distribution of innovations to their long-term earnings prospects while employed workers draw from a positively skewed distribution. Therefore as workers flow in and out of employment over the business cycle they face different distributions of risk to their earnings. Finally, one of the aggregate shocks that drives the model is a risk shock that alters the distribution of earnings risks that workers face conditional on employment status.

Section 3 chooses parameters of the model's income process and makes the case for cyclical shifts in earnings risk. The model is able to closely match both the cyclicality of earnings losses after displacement reported by Davis and von Wachter (2011) and the pro-cyclical skewness of earnings growth rates reported by Guvenen et al. (2014). These features of the data cannot be explained by changing unemployment dynamics themselves. For both of these pieces of evidence, the model is only able to match the data by including cyclicality in the distribution of earnings changes conditional on employment status. The analysis reported in this section includes constructing a quarterly time series for risk shocks that best fits the observed distributions of earnings growth rates reported by Guvenen et al. (2014) while being consistent with the results from Davis and von Wachter (2011).

Time-varying idiosyncratic risk has a quantitatively substantial effect on the dynamics of aggregate consumption. In particular, the standard deviation of quarter-to-quarter aggregate consumption growth is 46 percent larger than it is in a complete markets version of the model. Fifteen percentage points of this difference are due to the idiosyncratic risk shock that makes earnings growth rates negatively skewed in recessions. Thriteen percentage points arise as displaced workers and employed workers, respectively, draw from left-skewed and right-skewed distributions or earnings risk and the composition of the labor force shifts towards displaced workers in recessions. Simple unemployment risk, not including any associated effect on long-term earnings prospects, explains most of the remaining difference.

Non-durable and services consumption declined by 3.6 percent between the NBER peak and the the first quarter of 2009. At the same time, the Great Recession brought about a large spike in risk to individual earnings according to the series for risk shocks developed in Section 3. Using the model to simulate the consumption response to the deterioration of labor market conditions in the Great Recession predicts a 2.8 percentage point drop as a result of these labor market risks without any shock to the average earnings of employed workers. Furthermore, 1.2 percentage points of this drop are due to the risk shock and 0.4 percentage points come from the skewed distributions of earnings risk faced by displaced and employed workers in conjunction with the rise in the job-separation rate and fall in the job-finding rate.

While the model predicts that idiosyncratic risk lead to large changes in aggregate consumption during the Great Recession, it also predicts that the cross-sectional dispersion of consumption remained almost completely unchanged. This prediction is in line with the empirical findings of Perri and Steinberg (2012). This outcome reflects the fact that the increase in risk is short-lived and does not accumulate into a substantial change in the cross-sectional distribution of income and consumption levels. These results demonstrate that evidence of a constant cross-sectional distribution does not preclude the possibility that idiosyncratic risk and distributional shocks play an important role in aggregate dynamics.

The double-dip recession of the early 1980's shows a similar pattern to the Great Recession with a sharp drop in aggregate consumption. The baseline model also does a good job of accounting for this change in consumption as a result of changes in idiosyncratic risk. At other times in the sample 1977 to 2011, the predictions of the baseline model are roughly similar to a complete markets benchmark so it appears that the consumption dynamics emphasized here are particularly relevant in deep recessions.

Krusell and Smith (1998) show the business cycle dynamics of a heterogeneous agent version of the neoclassical growth model are generally close to those of the representative agent economy; however, when the model matches the distribution of net worth, aggregate consumption shows an increased correlation with aggregate income. This result reflects the greater sensitivity of consumption to income among low wealth individuals.

Market incompleteness can affect aggregate consumption through an alternative channel besides hand-to-mouth behavior. In particular, if the uninsurable risk that individuals face varies over time, there will be a time-varying precautionary savings motive that can generate additional fluctuations in aggregate consumption that are disconnected from aggregate income. The existing literature that studies the contribution of uninsurable idiosyncratic income risk to the business cycle has focussed on fluctuations in the unemployment rate as a source of time-varying uninsurable risk (Krusell and Smith, 1998; Challe and Ragot, 2016; Ravn and Sterk, 2013; Challe et al., 2015; Krueger et al., 2015). However, as unemployment is generally a short-lived shock to earnings, it is easily smoothed through self-insurance and only individuals with very low levels of savings will alter their consumption behavior in a meaningful way when the unemployment risk changes. One response is to make unemployment more painful by limiting the self insurance that individuals have. Challe and Ragot (2016), Ravn and Sterk (2013), and Challe et al. (2015) pursue this approach by calibrating their models such that the majority of individuals have little or no wealth. This approach could be justifiable on the grounds that much of household wealth takes the form of illiquid assets that cannot easily be used for consumption smoothing.

The contribution of this paper is to introduce a rich income process that incorporates cyclical variation in the risks to long-term earnings prospects as documented by Guvenen et al. (2014) and Davis and von Wachter (2011). Because these earnings shocks are persistent, they are more difficult to self-insure and even individuals with large amounts of savings will respond to changes in the distribution of earnings risks. Therefore, this model is able to match the distribution of net worth and still predict substantial changes in the dynamics of aggregate consumption due to changes in the precautionary savings motive.

The nature and source of cyclical changes in the earnings process are still somewhat poorly understood.[1] This paper gives a particular interpretation to the facts on the distribution of earnings changes in expansions and recessions—these changes in income are uninsured and unforeseen risks—and then goes on to consider the implications for aggregate consumption dynamics. Other implications of this type of risk have also been studied. For example, Constantinides and Duffie (1996), Storesletten et al. (2007), Constantinides and Ghosh (2014), and Schmidt (2016) investigate the asset pricing implications of this type of risk. The welfare cost of business cycles in the presence of these risks have been analyzed by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007). However, it does not appear that the consequences for the business cycle dynamics of aggregate quantities have been studied in the literature.

This paper is also related to the recent literature that investigates the role of uncertainty in business cycle fluctuations. In particular, Basu and Bundick (2012), Leduc and Liu (2016), and Fernández-Villaverde et al. (2015) emphasize the precautionary savings effect that follows an increase in uncertainty surrounding aggregate conditions such as preferences, technology, or taxes. In contrast to those studies, the focus here is on cyclical variation in microeconomic uncertainty faced by heterogeneous households. Other studies analyze the impact of cyclical microeconomic uncertainty faced by firms. This work is motivated by evidence of countercyclical dispersion in firm-level productivity, sales growth rates, and other measures of business conditions.[2] Bloom (2009), Bloom et al. (2012), and Bachmann and Bayer (2013) study the interaction of this microeconomic uncertainty with non-convex adjustment costs for investment and hiring. Arellano et al. (2016) and Gilchrist et al. (2014) explore the interaction

[1] Davis and von Wachter (2011) show that structural models of the labor market have difficulty explaining the size and cyclicality of present-value earnings losses after job displacement. Huckfeldt (2016) shows that the earnings losses are concentrated among workers who switch occupations and presents a model of skill accumulation and occupation switching that performs better but still struggles to explain the strong cyclicality. Jarosch (2015) shows that job insecurity at the bottom of the job ladder can explain long-term earnings losses for displaced workers, but he does not address the cyclicality of these earnings losses. This line of work has not sought to explain the pro-cyclical skewness of earnings growth rates, but the results presented here are reason to believe that the two phenomena are linked. Hubmer (2016) shows that a job ladder model can explain the skewness of earnings growth rates, but does not investigate the cyclicality of skewness.

[2] See Bloom (2014) for a review of the evidence.

of firm-level risks and financial frictions. This paper contributes to this literature by studying the importance of variations in the microeconomic uncertainty surrounding household incomes for aggregate consumption.

The paper is organized as follows: Section 2 presents the model. Section 3 describes the calibration and fit of the income process. Section 4 discusses the choice of other parameters and Section 5 describes the solution methods. Section 6 presents the results on the impact of time-varying earnings risks on the dynamics of aggregate consumption. The paper concludes with Section 7.

2 Model

I analyze a general equilibrium model with heterogeneous households and aggregate uncertainty. At the aggregate level, the model is similar to that of Krusell and Smith (1998). At the microeconomic level, the model incorporates time-varying idiosyncratic risk with an income process that captures the cyclical nature of idiosyncratic risk described by Davis and von Wachter (2011) and Guvenen et al. (2014).

2.1 Population, preferences and endowments

The economy is populated by a unit mass of individuals who survive from one period to the next with probability $1-\omega$ and each period a mass $\omega \in (0,1)$ of new agents is born, leaving the population size unchanged. At date 0, an individual seeks to maximize preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1-\omega)^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where C_t is the individual's consumption in period t. Individuals differ in their rates of time preference, which allows the model to generate additional heterogeneity in wealth holdings.

Individuals can be either employed, short-term unemployed, or long-term unemployed, and they transition between these states exogenously. Let λ^s and λ^ℓ be the job-finding rates for short-term and long-term unemployed and let ζ be the separation rate from employment to short-term unemployment. All separated workers initially enter short-term unemployment and if they fail to find a job in the first period of unemployment they then become long-term unemployed. These transition rates vary through time with aggregate conditions.

If employed, an individual exogenously supplies $\exp\{y\}$ efficiency units of labor, where y is the individual's (log) efficiency. The cross-sectional dispersion in earnings could be due to differences in wage or due to differences in hours. For lack of a better term, consider y to be "skill." This skill evolves according to

$$y = \theta + \xi,$$
$$\theta' = \theta + \eta',$$

where ξ is a transitory shock distributed $N(\mu_\xi, \sigma_\xi)$. The constant parameters of the distribution for ξ satisfy $\mathbb{E}[e^\xi] = 1$. η is a permanent shock to the individual's skill. Assuming that this shock is permanent as opposed to persistent has the advantage that it allows one state variable to be eliminated from the consumption-savings problem as described in Appendix C.[3] This type of income process is known to fit longitudinal earnings data well as shown by MaCurdy (1982) and Abowd and Card (1989).

The permanent shock, η, can be drawn from one of three distributions. Most workers draw from a distribution $N(\mu_{1,t}, \sigma_{\eta,1})$, which should be interpreted as a "standard" earnings change. There is considerable evidence that displaced workers sometimes experience large and persistent earnings losses (Jacobson et al., 1993; Couch and Placzek, 2010; Davis and von Wachter, 2011). In the model, conditional on an employment-unemployment transition a worker has a probability p_2 of drawing from $N(\mu_{2,t}, \sigma_{\eta,2})$ with $\mu_{2,t} < \mu_{1,t}$. With the remaining probability $1 - p_2$ these workers draw from the standard distribution centered at $\mu_{1,t}$. In line with the findings of Davis and von Wachter (2011), the extent of these long-term earnings losses can vary over time. Specifically

$$\mu_{1,t} = \bar{\mu}_t \tag{1}$$
$$\mu_{2,t} = \bar{\mu}_t + \mu_2 - x_t. \tag{2}$$

Here $\mu_2 < 0$ is a "typical" earnings loss that can be larger or smaller according to the latent

[3] See also Carroll et al. (2013).

factor x_t. $\bar{\mu}_t$ is a normalization such that $\mathbb{E}[e^{\eta}] = 1$ in all periods, which is without loss of generality given that there is also an aggregate component to wages as described below. This normalization disconnects "distributional" shocks from first-moment shocks.

Similarly, a fraction p_3 of employed workers draw from $N(\mu_{3,t}, \sigma_{\eta,3})$ with $\mu_{3,t} > \mu_{1,t}$. This component of the model captures the up-side risks that workers face, such as job promotions. The magnitude of up-side risks fluctuates over the cycle in the same way the down-side risks do

$$\mu_{3,t} = \bar{\mu}_t + \mu_3 - x_t. \tag{3}$$

The latent risk factor x_t generates pro-cyclical skewness in earnings changes by shifting the tails of the earnings growth rate distribution to the right and left.[4]

The model includes three sources of time-variation in idiosyncratic risk. First, the job-finding and separation rates vary, which even in the absence of a long-term earnings shock results in foregone earnings during the unemployment spells of varying incidence and duration. Second, as the separation and employment rates vary so do the chances of experiencing a long-term earnings loss or long-term earnings gain. Finally, the average size of long-term earnings losses and gains vary over time with the risk factor x_t.

The total labor input is $\bar{L} \equiv \mathbb{E}\left[e^{y} I_n\right]$, where I_n is an indicator for being employed. It is important that the model includes mortality risk as this allows for a finite cross-sectional variance of skills despite the fact that innovations to skills are permanent. When an individual dies, he or she is replaced by a newborn with no assets and $\theta = 0$. The unemployment rate among newborns is the same as prevails in the surviving population at that date. An individual's rate of time preference is fixed throughout his or her life and drawn initially from a stable two-point distribution.

[4]The model assumes that the shocks drawn between periods t and $t+1$ depend on the distributions centered around $\mu_{1,t}$, $\mu_{2,t}$, and $\mu_{3,t}$, which implies workers know the distribution of risk that they face between one period and the next.

2.2 Technology, markets, and government

A composite good is produced out of capital and labor according to

$$\bar{Y} = e^{z}\bar{K}^{\alpha}\bar{L}^{1-\alpha}$$

where z is an exogenous total factor productivity (TFP) and aggregate quantities are denoted with a bar. Capital depreciates at rate δ and evolves according to

$$\bar{C} + \bar{K}' = \bar{Y} + (1-\delta)\bar{K}.$$

The factors of production are rented from the households each period at prices that satisfy the representative firm's static profit maximization problem

$$W = (1-\alpha)e^{z}\bar{K}^{\alpha}\bar{L}^{-\alpha} \tag{4}$$

$$\tilde{R} = \alpha e^{z}\bar{K}^{\alpha-1}\bar{L}^{1-\alpha} + 1 - \delta. \tag{5}$$

Here $\tilde{R}$ is the return on capital and W is the wage paid per efficiency unit. Individuals save in the form of annuities with return $R \equiv \tilde{R}/(1-\omega)$, which exceeds the return on capital because the annuity pays only conditional on survival. I assume that savings must be non-negative due to borrowing constraints. Given the income process, in which the shocks to log-income are unbounded, the zero borrowing limit is the natural borrowing limit.

The Social Security Administration data used by Davis and von Wachter (2011) and Guvenen et al. (2014) refer to pre-tax earnings. As the model is calibrated to match facts from these papers it is a model of pre-government earnings. Taxes and transfers provide important insurance against idiosyncratic risks and so they are explicitly incorporated into the model. Let the net tax payment of an employed individual with earnings We^{y} be $We^{y} - (1-\tau)We^{(1-b^{y})y}$. The parameters τ and b^{y} control the level and progressivity of the tax, respectively. With $b^{y} = 0$, the tax system is linear. With $b^{y} = 1$ all individuals receive the same after-tax income. For incomes less than $(1-\tau)^{1/b^{y}}$ the average tax rate is negative and the individual receives a transfer from the government. This parametric form for a progressive tax system has been used in the literature by Feldstein (1969), Benabou (2000), Heathcote et al. (2014) and others.

Unemployed workers receive taxable unemployment insurance payments with a post-tax replacement rate $b^{u,s}$ for short-term unemployed and $b^{u,\ell}$ for long-term unemployed. I assume the level of the tax system, τ, is adjusted period by period to balance the budget of the tax and transfer system.

2.3 Aggregate shock processes

TFP evolves according to

$$z' = \rho_z z + \epsilon'_z. \tag{6}$$

In the labor market, aggregate shocks occur at the start of a period and employment outcomes in period t reflect the shocks realized at date t. The job-finding rate for short-term unemployed and job-separation rate are modeled as AR(1) processes with correlated innovations. Specifically,

$$\hat{\lambda}^{s\prime} = (1-\rho_\lambda)\hat{\lambda}^{s*} + \rho_\lambda \hat{\lambda}^{s} + \epsilon'_\lambda, \tag{7}$$

$$\hat{\zeta}' = (1-\rho_\zeta)\hat{\zeta}^{*} + \rho_\zeta \hat{\zeta} + \epsilon'_\zeta, \tag{8}$$

where hats denote the inverse-logistic transformation[5]. The job-finding rate of long-term unemployed is a scaled version of the short-term job-finding rate given by $\lambda^\ell_t = (\lambda^{\ell*}/\lambda^{s*})\lambda^s_t$. The constant parameters λ^{s*}, $\lambda^{\ell*}$, and ζ^* determine the mean job-finding and separation rates. This formulation captures some of the decline in job-finding rates with the duration of an unemployment spell without introducing any additional state variables or shocks. The process for skill risk, x, follows

$$x' = \rho_x x + \epsilon_x. \tag{9}$$

The model allows for correlation among ϵ_λ, ϵ_ζ, and ϵ_x because they are all closely related to developments in the labor market. For simplicity, ϵ_z follows an independent process.

[5] That is, ζ and $\hat{\zeta}$ are related according to $\zeta = 1/(1+e^{-\hat{\zeta}})$.

2.4 The individual's decision problem

The idiosyncratic state variables of the consumption-savings problem are cash on hand, call it A, permanent skill, θ, and employment status, m, which takes three values to distinguish employed, short-term unemployed and long-term unemployed. Individuals also differ in their rates of time preference although these are not state variables as they are fixed within a lifetime. The aggregate states are $S \equiv \{z, \lambda, \zeta, x, \Gamma\}$, where Γ is the distribution of individuals over the state space from which one can calculate aggregate capital, $\bar{K}$, and the aggregate labor supply. Appendix C describes how the model can be normalized to eliminate some of these state variables. The decision variable is end-of-period savings, K'.

The decision problem is then

$$V(A, \theta, m, S) = \max_{K' \geq 0} \left\{ \frac{(A - K')^{1-\gamma}}{1-\gamma} + \beta(1-\omega)\mathbb{E}\left[V(A', \theta', m', S')\right] \right\}$$

subject to

$$A' = R(S')K' + (1 - \tau(S'))W(S')e^{(1-b^y)(\theta+\eta'+\xi')}n(m'),$$

where $n(m) = 1$ for employed, $n(m) = b^{u,s}$ for short-term unemployed, and $n(m) = b^{u,\ell}$ for long-term unemployed. The law of motion for the aggregate state is given by (6), (7), (8), (9), and a law of motion for the distribution of idiosyncratic states, $\Gamma' = H_\Gamma(S, \zeta', \lambda')$.[6]

2.5 Equilibrium

Let $F(A, \theta, m, S)$ be the optimal decision rule for K' in the household's problem. Aggregate savings are

$$\bar{K}' = \int F(A, \theta, m, S)\, d\Gamma(A, \theta, m), \tag{10}$$

Given a set of exogenous stochastic processes for z, ζ, λ, and x, a recursive competitive equilibrium consists of the law of motion for the distribution, H_Γ, value function, V, and policy rule, F, and pricing functions W and R. In an equilibrium, V and F are optimal for

[6] H_Γ depends on ζ' and λ' because Γ' is the distribution of individuals after employment transitions have occurred.

the household's problem, $R = \tilde{R}/(1-\omega)$, $\tilde{R}$ and W satisfy (4) and (5), and H_Γ is induced by F and the idiosyncratic income process.

3 The income process and cyclical risks

I use a simulated method of moments procedure to choose parameters for the income process to fit both the level and cyclicality of earnings losses conditional on displacement reported by Davis and von Wachter and the pro-cyclical skewness of the earnings growth rate distribution reported by Guvenen et al. (2014). Simulating the income process requires empirical counterparts to ζ_t, λ_t^s, λ_t^ℓ, x_t, and W_t. ζ_t, λ_t^s, and λ_t^ℓ are constructed from observed stocks of short-term, medium-term, and long-term unemployed as explained in B. While the Guvenen et al. data is available at an annual frequency, business cycles are typically analyzed at the quarterly frequency. Therefore I use the Guvenen et al. data to construct a quarterly time series for x_t by simulating quarterly data and aggregating to annual observations. An assumption underlying this approach is that developments in the labor market drive both x_t and observable indicators of labor market conditions that are available at a quarterly frequency. The procedure uses four such indicators: the ratio of short-term unemployed (fewer than 15 weeks) to the labor force, the same ratio for long-term unemployed (15 or more weeks), an index of average weekly hours, and the labor force participation rate. x_t is then assumed to be a linear combination of these four series with factor loadings to be determined.

Simulating the income process also requires a time series for the aggregate wage per efficiency unit of labor, W_t. In the model this is determined by the marginal product of labor, but the simulation procedure uses an empirical estimate of average income growth so as to avoid the computationally costly process of solving and simulating the full model of the economy. Just as x_t is a linear combination of observed labor market indicators, W_t is also a linear combination of (i) the percent change in per capita wage and salary income, (ii) the short-term job-finding rate, (iii) the estimate of x_t constructed above, and (iv) a constant with the four factor loadings to be determined in the search over parameters.

In addition to the factor loadings for x_t and W_t, the procedure also searches for values for p_2, p_3, μ_2, μ_3, $\sigma_{1,\eta}$, $\sigma_{2,\eta}$, $\sigma_{3,\eta}$, and σ_ξ while imposing the restriction $\sigma_{2,\eta} = \sigma_{3,\eta}$.

Symbol	Description	Value
μ_2	Mean of left tail of η distribution	-0.1669
μ_3	Mean of right tail of η distribution	0.3940
$\sigma_{1,\eta}$	St. dev. of center of η distribution	0.0638
$\sigma_{2,\eta}$	St. dev. of right tail of η distribution	0.3337
$\sigma_{3,\eta}$	St. dev. of left tail of η distribution	0.3337
σ_ξ	St. dev. of transitory income shock	0.1545
p_2	Prob. earnings loss at layoff	0.7145
p_3	Prob. earnings jump	0.0189

Table 1: Parameters of the income process.

For each candidate parameter vector, the procedure simulates the income process for a panel of individuals including employment and mortality shocks and evaluates an objective function that penalizes the distance between the model-implied moments and the empirical moments. The target moments are the year-by-year values for the median, 10th percentile and 90th percentile of the one-year, three-year and five-year earnings growth distributions. The Guvenen et al. data range from 1978 to 2011. The level and cyclicality of the mean long-run earnings losses conditional on layoff as reported by Davis and von Wachter (2011) serve as additional target moments. I target an earnings loss of 10 percent for NBER expansions and a loss of 20 percent for recessions based on the long-run responses in their Figure 4. B contains further discussion of the implementation of this method.

Table 1 shows the resulting parameter values. Figure 1 shows the model's fit to the earnings growth distribution at one-year, three-year and five-year horizons. The model does a good job of matching the moments of the three-year and five-year earnings changes. While the model fails to generate the volatility of the 10th and 90th percentiles for one-year changes, this is not too worrisome as the three-year and five-year earnings changes are a better reflection of long-term earnings risks that are of particular interest here.

Figure 2 shows the time series for x_t that is generated by this procedure.[7] One can see that there are sharp spikes in this measure of idiosyncratic risk during recessions. The correlation of this series with initial claims for unemployment insurance is 0.67. The finding that idiosyncratic risk is closely related to initial claims for unemployment insurance is supported by the

[7] The time series shown in this figure is available from the author's website.

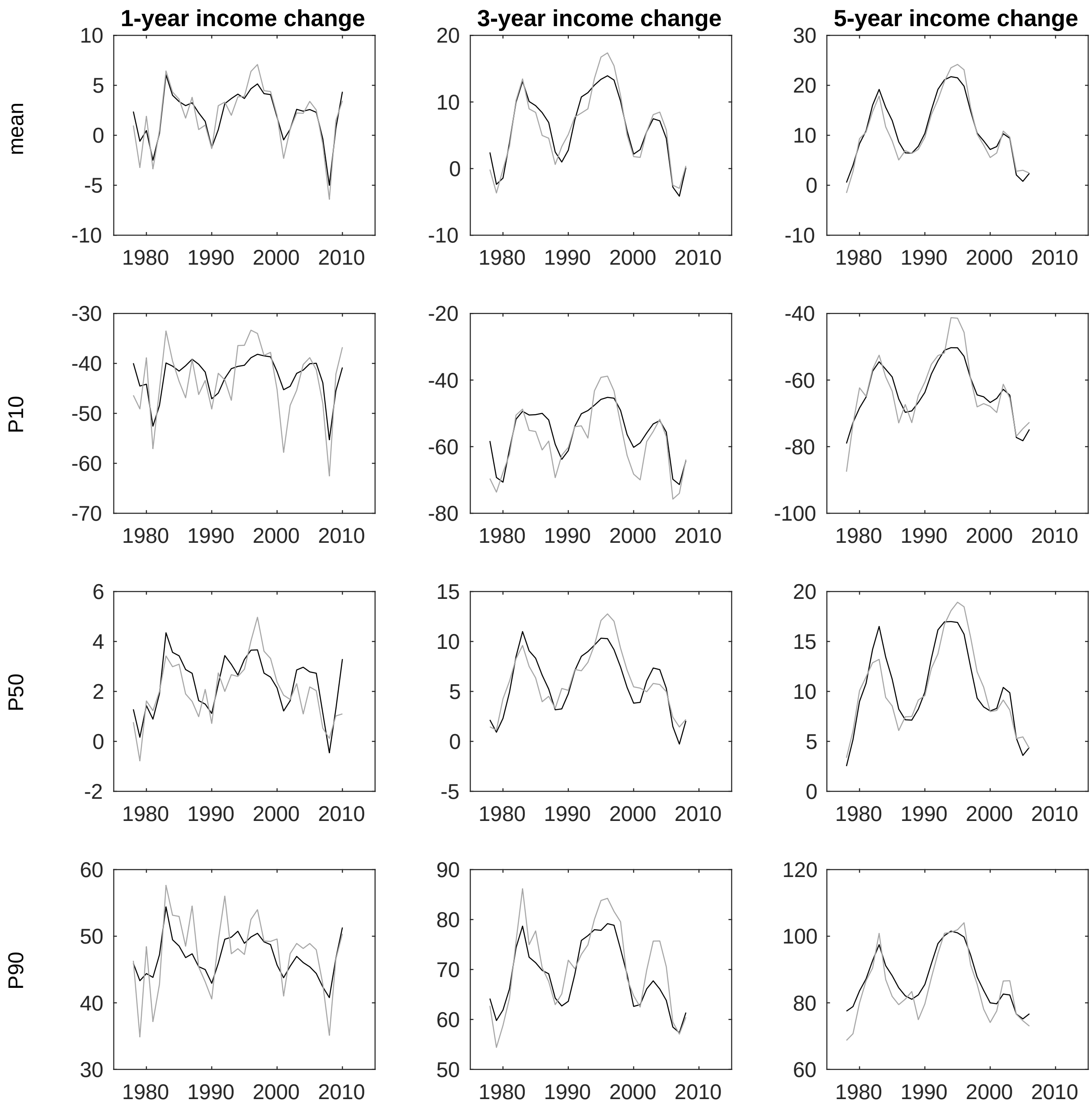

Figure 1: Simulated (dark line) and empirical (light line) moments of the income process.

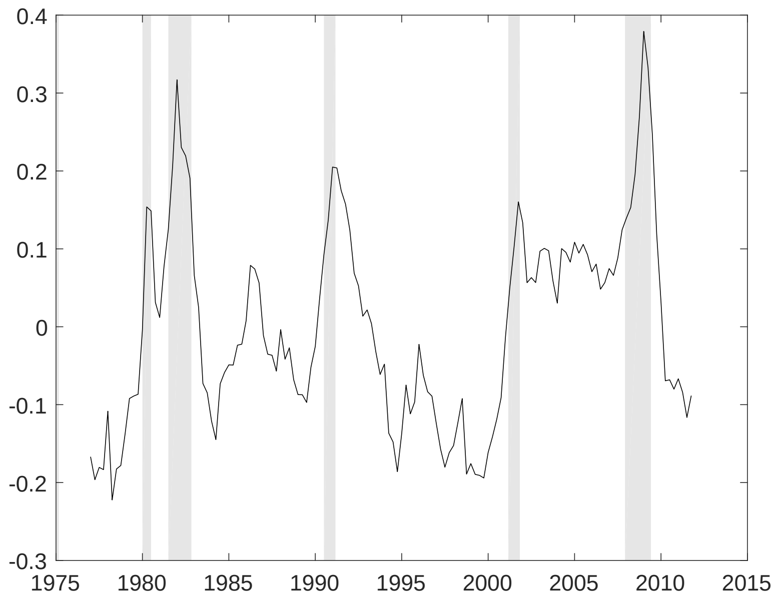

Figure 2: Empirical measure of x_t.

work of Schmidt (2016). Schmidt (2016) also creates a quarterly-time series of idiosyncratic risk based on the Guvenen et al. (2014) data. He uses annual observations of the skewness of earnings growth rates and then interpolates this skewness index to a quarterly time series for the skewness of idiosyncratic shocks using 109 macroeconomic time series. He also finds that the resulting skewness index is closely related to initial claims for unemployment insurance.

Figure 3 shows a measure of skewness in the five-year earnings changes for the model and the data. Kelley's skewness is Guvenen et al.'s preferred measure of skewness because it is less sensitive to extreme observations than the third central moment. Kelley's skewness is calculated from the 10th, 50th and 90th percentiles of the distribution as $((P90 - P50) - (P50 - P10))/(P90 - P10)$. The model slightly understates the volatility in this measure of risk.

The estimated income process does a good job matching the cyclicality in the average earnings losses after displacement. In the estimated model, the average long-run earnings loss for a displacement that occurs in a recession is 24.1 percent while it is 9.5 percent for expansions. These earnings losses are close to the targets of 20 percent and 10 percent, respectively.

The model's ability to match the cyclicality in the long-run earnings losses after displacement requires movements in x_t as with a constant x_t the earnings loss would be constant. As

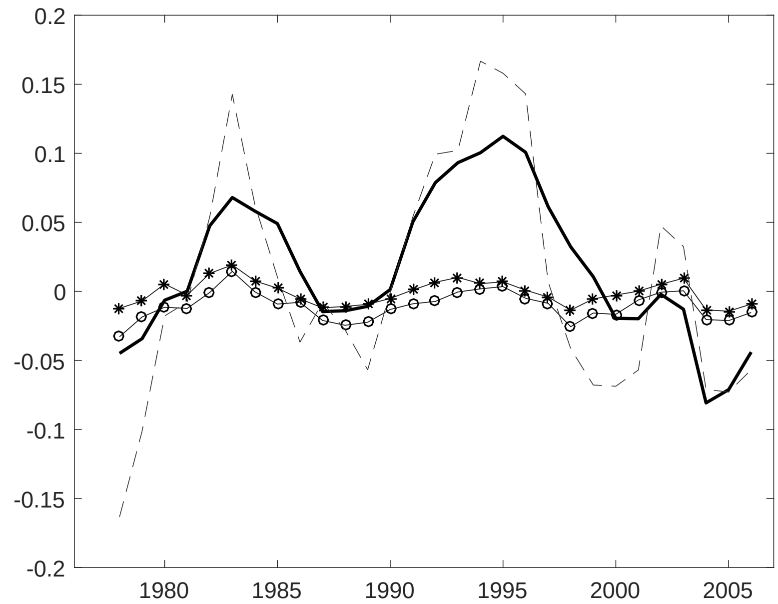

Figure 3: Kelley's skewness of five-year earnings growth rates for data (dashed line), baseline model (thick line), with $p_2 = p_3 = 0$ (stars), with $x_t = 0\ \forall t$ (circles).

I demonstrate next, the model's ability to generate pro-cyclical skewness in earnings growth rates also depends strongly on movements in x_t. This is not an obvious finding because there are two other sources of cyclical risk that could in principle generate pro-cyclical skewness: (i) cyclical unemployment risk and (ii) a constant earnings loss at displacement together with a counter-cyclical separation rate. Consider two restricted versions of the income process. In the first case set $p_2 = p_3 = 0$ so long-term earnings risk is independent of employment status and independent of x_t. In the second case, set $x_t = 0$ for all t so there is a chance of long-term earnings losses associated with job loss and a chance of a large earnings gain while employed but the extent of these earnings changes does not vary over time. Figure 3 shows the results of recalibrating the model under these restrictions. Neither of these alternative models is able to generate much cyclical variation in the skewness of five-year earnings growth rates. Based on this analysis of alternative specifications I conclude that the time-varying magnitude of idiosyncratic risks captured by variation in x_t is a crucial part of the explanation of the observed pro-cyclical skewness in earnings growth rates as well as the cyclicality of earnings losses after displacement.

Most of the variation in Kelley's skewness that the model generates comes from variation in

x_t and very little comes from time varying unemployment risk.[8] Blass-Hoffmann and Malacrino (2016) have recently argued that unemployment risk can explain the pro-cyclical skewness of earnings growth rates as measured by the third-central moment. I have found that to be the case for the third central moment but not for Kelley's skewness. Time-varying unemployment risk leads to fluctuations in the number of workers who are close to full-year unemployed, which leads to fluctuations in the mass of workers in the far left tail of the earnings growth distribution and this has a powerful effect on the third central moment. Kelley's skewness is by construction not sensitive to outliers and is relatively little affected by these forces.

4 Calibration

To parameterize the aggregate shock processes, AR(1) processes are estimated for the three series $\hat{\zeta}_t$, $\hat{\lambda}^s_t$, and x_t, which are constructed as in Section 3. A Cholesky decomposition is applied to the covariance matrix of residuals yielding the following system

$$\left[\hat{\zeta}' - \hat{\zeta}^*, \hat{\lambda}^{s\prime} - \hat{\lambda}^{s*}, x'\right]^T = D\left[\hat{\zeta} - \hat{\zeta}^*, \hat{\lambda} - \hat{\lambda}^{s*}, x\right]^T + \epsilon',$$

where $\hat{\zeta}^* = -\log(1/0.0453 - 1)$, $\hat{\lambda}^{s*} = -\log(1/0.7874 - 1)$,[9] D is a diagonal matrix with diagonal elements $[0.9363, 0.9240, 0.9222]$ and the decomposed covariance matrix of ϵ is

$$\begin{pmatrix} 0.0428 & 0 & 0 \\ -0.0380 & 0.0778 & 0 \\ 0.0331 & -0.0017 & 0.0345 \end{pmatrix}.$$

The steady state value of the job-finding rate for long-term unemployed is set to $\lambda^{\ell*} = 0.458$, which is the mean of the series constructed in Section 3. The coefficient of relative risk aversion is set to 2, the depreciation rate is set to 2 percent per quarter. The persistence of the productivity process is set to 0.96 in line with typical estimates for the US. The labor share

[8] Appendix G shows the results of an alternative specification in which unemployment risk is constant over time and in this case the dynamics of Kelley's skewness are very similar to those produced by the baseline model.

[9] Recall that hatted variables are inverse-logistic transformations so the steady state values of ζ and λ^s are 0.0453 and 0.7874, respectively.

is set to 64 percent and the mortality risk is set to 0.5 percent per quarter for an expected working lifetime of 50 years.

The unemployment insurance replacement rates, $b^{u,s}$ and $b^{u,\ell}$, are set to 0.58 and 0.33, respectively, in line with the income dynamics of unemployment insurance recipients reported by Rothstein and Valletta (2014).[10] The skill insurance parameter b^y is set to 0.151, which is the progressivity of the tax-and-transfer system estimated by Heathcote et al. (2014) to fit the relationship between pre- and post-government income in PSID data.[11]

There are two values of β in the population with 80 percent of the population having the lower value and 20 percent having the higher value. These values and the volatility of the productivity process are set to match the following moments in an internal calibration: an annual capital-output ratio of 3.32, a wealth share of the top 20 percent by wealth equal to 83.4 percent of total wealth (see Diaz-Gimenez et al., 2011), and a standard deviation of log output growth equal to 0.0084. The resulting standard deviation of TFP innovations is 0.0081 and the discount factors are 0.967 and 0.990, respectively.

The model generated distribution of wealth appears in Table 2. The baseline model does an excellent job of matching the data all along the Lorenz curve including the holdings of the very rich. That the model can generate extremely wealthy households is partially due to preference heterogeneity as shown by the comparison with the second row of the table in which all individuals have the same rate of time preference. Even without preference heterogeneity, however, some individuals accumulate large wealth positions by virtue of good luck in their

[10]Refer to Figure 7 in Rothstein and Valletta (2014) and omit the earnings of other household members. Most of the income after unemployment benefits are exhausted comes from means-tested transfers and Social Security. In the calibration unemployment benefits are received for the first period of unemployment and not for subsequent periods. As the model period is one quarter, this approach understates the generosity of the unemployment insurance system. On the other hand, many non-employed individuals do not receive any unemployment benefits (see Rothstein and Valletta Table 1).

[11]The Social Security Administration data are at the level of the individual worker rather than household so another consideration is insurance provided within families. One could calibrate the social insurance system to be more generous to account for other forms of insurance but I do not pursue that because it is difficult to quantify the extent of this additional insurance.

Heathcote et al. (2014) discuss the fact that the tax-and-transfer system became more progressive during the Great Recession. Whether or not this time-varying insurance is important depends on how constrained households are. If they are unconstrained, the precautionary savings motive is driven by changes in the households entire future earnings path. As the shocks to earnings that arise during the recession have long lasting effects, what is particularly relevant is the degree of insurance over the household's remaining lifetime as opposed to the progressivity of the system at a point in time. However, if a substantial portion of households are constrained, the degree of insurance at a point in time could be important in that transfers have a strong effect on current consumption. The analysis assumes a constant tax-and-transfer system for simplicity.

	Share of wealth by quintile					and held by richest			Gini
	1st	2nd	3rd	4th	5th	10%	5%	1%	
Baseline	0.01	0.02	0.04	0.09	0.84	0.70	0.55	0.32	0.78
Common-β	0.04	0.07	0.10	0.18	0.62	0.46	0.35	0.18	0.57
Data	0.00	0.01	0.05	0.11	0.83	0.71	0.60	0.34	0.82

Table 2: Distribution of wealth. Data refer to net worth from the 2007 Survey of Consumer Finances as reported by Diaz-Gimenez et al. (2011).

income draws coupled with a strong precautionary motive. In this regard the model has some similarity to that of Castaneda et al. (2003) where large wealth positions result from large income shocks. The model implies a distribution of earnings that displays a similar level of dispersion to what is found in the data: the Gini index for earnings is 0.63 as compared to 0.64 in the Survey of Consumer Finances (see Diaz-Gimenez et al., 2011).

5 Computation

The model presents two computational challenges. First, the aggregate state of the model includes the endogenous distribution of households over individual states. I solve the model with a version of the Krusell-Smith algorithm and replace this distribution with the first moment for capital holdings, $\bar{K}$, six summary statistics of the income distribution stored in a vector denoted Q, and the mass of short-term and long-term unemployed denoted by u^s and u^ℓ. The distribution of income evolves over time and the distribution is needed to calculate both the aggregate labor supply and the government's revenue from the progressive income tax. Appendix A explains which cross-sectional moments appear in Q and the law of motion for Q. The aggregate state is then $S_t = \{z, \hat{\zeta}, \hat{\lambda}^s, x, \bar{K}, \hat{u}^s, \hat{u}^\ell, Q\}$. The second computational challenge is the curse of dimensionality as the model includes 13 aggregate state variables,[12] three individual states and four aggregate shocks. To compute solutions to the individual's problem efficiently, my computational algorithm uses the method proposed by Maliar and Maliar (2015) to construct a grid on the part of the aggregate state space that the system actually visits. This approach reduces the computational cost of having many state variables

[12]As Q is a vector with dimension 6, S contains 13 continuous variables.

while still allowing for accurate solutions by avoiding computing the solution for combinations of states that are very unlikely to arise in practice. For individual cash on hand, the algorithm uses an endogenous grid point method that places 150 grid points on K'. Appendix D provides further discussion of the methods and presents several accuracy checks.

6 Results

I now assess the extent to which time-varying idiosyncratic risk alters the dynamics of aggregate consumption by comparing five economies that successively simplify the model: (i) the baseline model described above; (ii) a version of the model with stable distributions of earnings gains and losses over the cycle (i.e. $x_t = 0$ for all t); (iii) a version without large earnings gains or losses (i.e. $p_2 = p_3 = 0$); (iv) a version with $p_2 = p_3 = 0$ and a single rate of time preference so there is less wealth heterogeneity; and (v) a complete markets version of the model. In the complete markets model, households have a common discount rate and all shocks are insurable including mortality risk, which leads to the standard Euler equation for aggregate consumption

$$\bar{C}^{-\gamma} = \beta \mathbb{E}_t \left[\bar{C}'^{-\gamma} \tilde{R}' \right]$$

as shown in Appendix E. Each version of the model recalibrates the discount rate(s) to match the capital-output ratio and also the wealth holdings of the top 20% for versions of the model with heterogeneity in time preference.

6.1 Unconditional second moments

Table 3 displays standard deviations and correlations of output and consumption both in log-levels and in growth rates. The standard deviation of consumption growth is 46 percent larger in the baseline model than in the complete markets version of the model. To isolate the effects of risk shocks one can compare the first and second lines, which show the risk shock raises the volatility of consumption growth and reduces the correlation of consumption and income growth. The increase in consumption volatility is 15 percent of the complete markets volatility. Both the increase in consumption volatility and the decrease in the correlation of output and consumption growth reflect the additional source of consumption volatility that is imperfectly related to changes in aggregate income.

Even in the absence of risk shocks ($x_t = 0 \ \forall t$), individuals still face the possibility of large, but stable, earnings losses when they are displaced and the possibility of large earnings gains while employed. As workers flow in and out of unemployment, these risks can become more or less salient. Line (iii) of the table sets $p_2 = p_3 = 0$ to eliminate these risks and consumption volatility falls by 13 percent of the complete markets volatility. Line (iv) builds on line (iii) and turns off the heterogeneity in time preference. Doing so further reduces consumption volatility and accounts for about a quarter of the total difference between line (i) and line (v). Comparing lines (iv) and (v), one sees little effect of simply allowing for market incompleteness without incorporating any of the other components of the model, which confirms the original Krusell and Smith (1998) finding. Comparing lines (iii) and (v) shows that the effect of simple unemployment risk in a model with realistic wealth inequality is not insignificant. This finding echoes that of Krusell and Smith's stochastic-β economy and the work of Challe and Ragot (2016). However, the time-varying idiosyncratic risk elements introduced in lines (i) and (ii) more than double the effect of market incompleteness.

While consumption growth is more volatile when idiosyncratic risk is time-varying, the standard deviation of the level of consumption is only marginally affected. The level of consumption reflects low-frequency developments and is dominated by movements in the capital stock and TFP. As the extent of idiosyncratic risk appears to spike in recessions and quickly recede to more normal levels—as shown in Figure 2—its effects on the level of consumption are short lived. These high-frequency movements in aggregate consumption are much more visible in the standard deviation of consumption growth than they are in the volatility of the level of consumption.

Output is somewhat more stable in the baseline model than in the complete markets model. If market incompleteness pushes consumption down in a recession, this creates more investment and raises output in the next period. In this model these outcomes are direct implications of the aggregate resource constraint. One way of addressing this co-movement problem is to introduce nominal rigidities and constraints on monetary policy as in Basu and

		$\sigma_{\Delta\bar{Y}}$	$\sigma_{\Delta\bar{C}}$		$\sigma_{\bar{Y}}$	$\sigma_{\bar{C}}$	$\rho_{\Delta\bar{Y},\Delta\bar{C}}$	$\rho_{\bar{Y},\bar{C}}$
				Relative				
(i)	Baseline	0.816	0.422	1.458	3.104	2.368	0.847	0.938
(ii)	Constant risk	0.809	0.378	1.307	3.114	2.358	0.965	0.949
(iii)	No tails	0.812	0.339	1.172	3.196	2.282	0.988	0.945
(iv)	Common-β, no tails	0.811	0.308	1.065	3.251	2.197	0.990	0.934
(v)	Complete markets	0.815	0.289	1.000	3.293	2.194	0.982	0.928
(vi)	Data	0.845	0.520	1.770	4.353	2.982	0.540	0.919

Table 3: Standard deviations (σ) and correlations (ρ) of aggregate output ($\bar{Y}$) and consumption ($\bar{C}$) growth rates (denoted with Δ) and log-levels. Standard deviations are scaled by 100. Empirical moments for log-levels refer to real GDP and consumption of non-durables and services linearly detrended.

Bundick (2012).

6.2 The Great Recession

In order to illustrate the implications of time-varying risk, I now assess its contribution to the path of aggregate consumption during the Great Recession by simulating the economy's response to the observed labor market shocks. This analysis is not meant to be a full characterization of the Great Recession, for instance TFP remains constant in the simulation, but rather it is meant to illustrate the implications of these labor market developments for aggregate consumption.

The simulation proceeds as follows: the economy is in its risky steady state[13] in 2007:I and is then subject to shocks to the job-finding rate, the separation rate and the skewness of earnings shocks, x. The paths for the job-finding and separation rates are chosen so that the model generates the observed path for the unemployment rate and the number of unemployed with duration greater than three months. The path for x is the one from Section 3 and shown in Figure 2. This simulation can be conducted with each of the benchmark models considered in Table 3.

[13] Coeurdacier et al. (2015) define the risky steady state as the point to which the economy will converge if the realization of aggregate shocks is zero for all periods. This concept differs from the deterministic steady state in that the agents believe that aggregate shocks can occur and this contributes to their precautionary savings motive.

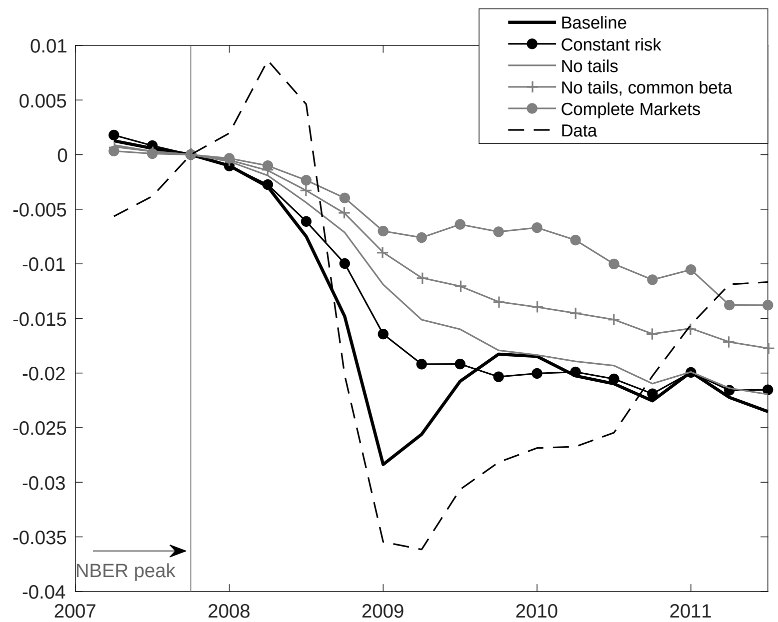

Figure 4: Dynamics of aggregate consumption implied by labor market shocks in the Great Recession. Data refer to per capita consumption of non-durables and services deflated with the GDP deflator and detrended with the HP filter with smoothing parameter 1600.

Figure 4 plots the path for consumption starting in 2007:II and normalized to one in 2007:IV, which was the peak of the expansion as defined by the NBER. In addition to the four versions of the model, the figure also plots the data on aggregate consumption of services and non-durable goods detrended with the HP filter.

In the data, consumption falls by 3.6 percent by 2009:I while the baseline model predicts a 2.8 percent decline. The complete markets model predicts a decline of 0.7 percent. In the absence of the risk shock, the model predicts a decline of 1.6 percent. The job-separation rate rose substantially during this period and workers who separate are exposed to the possibility of large long-run earnings losses. Turning off this channel and the associated up-side risk for employed workers reduces the predicted consumption decline to 1.2 percent. Over the course of 2009 the risk shock dissipates rapidly and aggregate consumption recovers somewhat, a pattern that is also evident in the data. The model predicts a prolonged slump in consumption that is substantially larger than what is predicted by the complete markets model. The additional decline in consumption in 2010 and beyond can be attributed to the elevated unemployment rate, which has a larger effect on consumption in the baseline model than in the version of the model with a single rate of time preference and fewer low-wealth households.

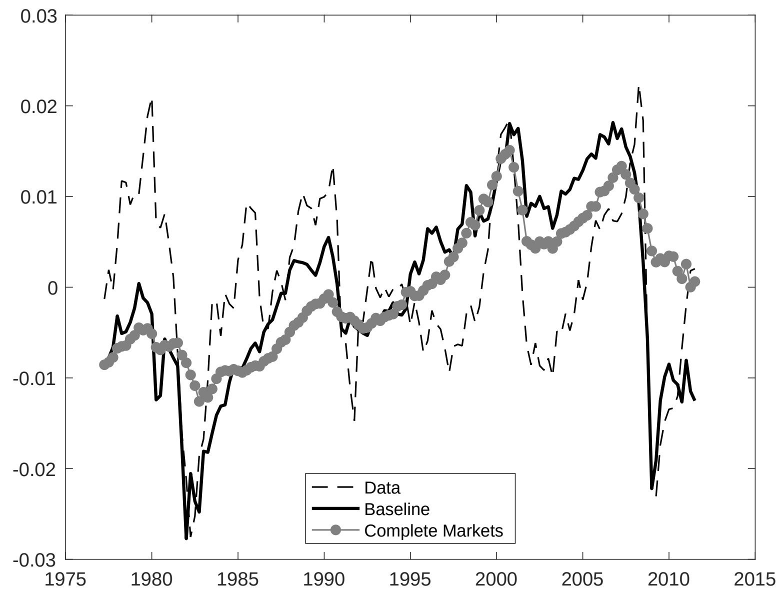

Figure 5: Dynamics of aggregate consumption implied by labor market shocks. Data refer to per capita consumption of non-durables and services deflated with the GDP deflator and detrended with the HP filter with smoothing parameter 1600.

Figure 5 repeats the same simulation over the time period 1977 to 2011. The baseline model predicts steep declines in consumption in the early 1980s and during the Great Recession and in each instance the model lines up well with the data. The complete markets model predicts much smaller consumption declines in these episodes. At other times, the baseline model and the complete markets model behave in relatively similar fashions. It appears that the forces emphasized here are particularly relevant for the behavior of aggregate consumption in deep recessions. In interpreting the figure, one should keep in mind that the simulation involves no change in TFP and as a result only modest changes in wages so it is not surprising that the model fails to replicate all of the movements in aggregate consumption.[14]

Perri and Steinberg (2012) document that there was little change in the cross-sectional distribution of disposable income and consumption during the Great Recession. One might ask whether the increase in income risk in my analysis creates a counterfactual increase in the dispersion of consumption in the cross section. Figure 6 shows that the model is consistent with the Perri-Steinberg observations by plotting the ratios of the 90th and 50th percentiles and of the 50th and 20th percentiles of consumption and of disposable income. The model

[14]Appendix G shows similar plots for output and investment and there one can see that the labor market shocks included in the simulation only account for a small amount of the fluctuations in aggregate income.

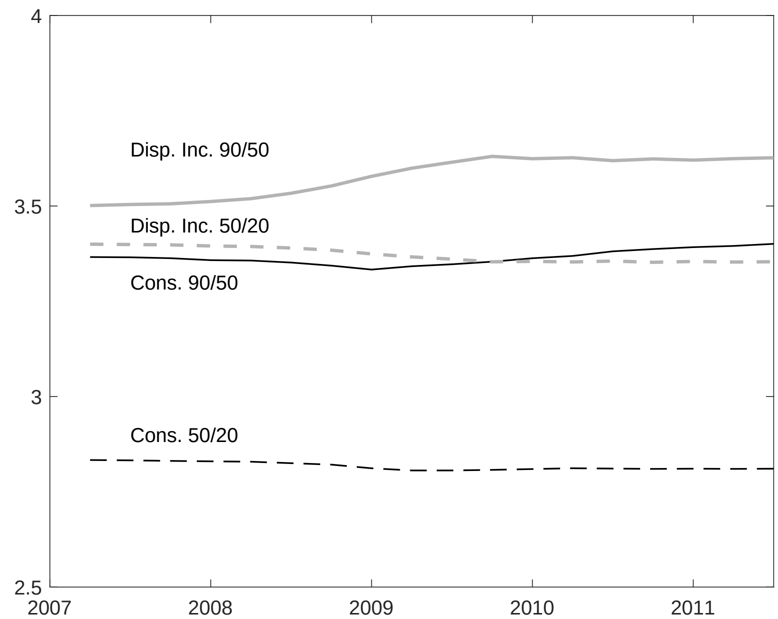

Figure 6: Cross-sectional distribution of levels of consumption and disposable income. Ratios of 90th and 50th percentiles and 50th and 20th percentiles.

predicts that these ratios are nearly constant as in the data.[15]

While the invariance of the consumption distribution can be partly attributed to insurance, either self insurance or social insurance, this cannot be the whole explanation as the increase in risk leads to a substantial drop in the average level of consumption. There are two explanations for how a change in risk can lead to a drop in aggregate consumption without affecting the cross-sectional distribution of consumption. First, the cross sectional distributions of income and consumption reflect the accumulation of past shocks and these distributions are only marginally affected by one quarter's innovations. As the aggregate shock to risk is short-lived it does not accumulate to substantial differences in the cross sectional distribution of levels. Second, the increase in risk leads all households to reduce their consumption more or less in unison as they all face more risk and self insurance is not very effective in smoothing the persistent income shocks so high-wealth households respond similarly to low-wealth households. As a result, the increase in idiosyncratic risk leads to a substantial drop in aggregate consumption with little effect on the cross sectional distribution of consumption.

[15]The right tail dispersion for disposable income does increase somewhat and this is also evident in Figure 4 from Perri and Steinberg (2012).

7 Conclusion

The deterioration of labor market conditions in the Great Recession has renewed interest in the effects of idiosyncratic risk on the business cycle. Changes in idiosyncratic risk will only have strong effects on consumption if individuals are not self-insured against these risks. This paper focusses on changes in the distribution of shocks to the persistent component of earnings. As these shocks are highly-persistent they are difficult to self-insure and even wealthy individuals are sensitive to changes in these risks. The results show that time-varying idiosyncratic risks substantially raises the volatility of aggregate consumption growth and can explain the sharp decline in aggregate consumption during the Great Recession.

This paper has focussed on the dynamics of aggregate consumption. At the aggregate level, the model is a version of the flexible-price real business cycle model with exogenous labor supply and as a result an increase in household savings necessarily leads to an increase in investment and an increase in output in future periods. Moreover, there is no endogenous feedback between the level of consumption and the extent of risk. Ravn and Sterk (2013) and Challe et al. (2015) explore an amplification mechanism that runs from unemployment risk to precautionary savings to reductions in aggregate demand and back to unemployment risk. This same chain of events could be triggered even more powerfully by the type of time-varying risk studied here. More generally, future work might incorporate cyclicality in the distribution of persistent earnings shocks as an important source of fluctuations in aggregate consumption in richer models of the business cycle.

Appendix

A Dynamics of Q

In order to compute the aggregate labor endowment we need to keep track of $\mathbb{E}\left[e^{y} I_{n}\right]$, where I_{n} is an indicator for employed. To compute the budget-clearing tax rate we need to keep track of $\mathbb{E}\left[e^{(1-b^{y})y} I_{n}\right]$. To compute the evolution of these moments, define $Q_{v,m}=\mathbb{E}\left[e^{vy}|m\right]$ for $v \in 1, 1-b^{y}$ and employment status m. As m can take three values, there are six $Q_{v,m}$ variables in total. We can then describe the moments of interest as follows

$$\mathbb{E}\left[e^{vy} I_{n}\right]=Q_{v,m=\text{employed}} \operatorname{Pr}[m=\text{employed}].$$

Moreover

$$\begin{aligned}
Q_{v,m} &= \mathbb{E}\left[e^{vy}|m\right] \\
&= \mathbb{E}\left[e^{v(\theta+\xi)}|m\right] \\
&= \mathbb{E}\left[e^{v\xi}\right] \mathbb{E}\left[e^{v\theta}|m\right]
\end{aligned}$$

as ξ is independent of θ and m. Continuing

$$\begin{aligned}
Q_{v,m}^{\prime} &= \mathbb{E}\left[e^{v\xi^{\prime}}\right] \mathbb{E}\left[e^{v\theta^{\prime}}|m\right] \\
&= \omega \mathbb{E}\left[e^{v\xi^{\prime}}\right]+(1-\omega) \mathbb{E}\left[e^{v\xi^{\prime}}\right] \mathbb{E}\left[e^{v\theta+v\eta^{\prime}}|m^{\prime}\right] \\
&= \omega \mathbb{E}\left[e^{v\xi^{\prime}}\right]+(1-\omega) \mathbb{E}\left[e^{v\xi^{\prime}}\right] \mathbb{E}_{m}\left[\mathbb{E}\left[e^{v\theta+v\eta^{\prime}}|m^{\prime}, m\right]|m^{\prime}\right] \\
&= \omega \mathbb{E}\left[e^{v\xi^{\prime}}\right]+(1-\omega) \mathbb{E}\left[e^{v\xi^{\prime}}\right] \mathbb{E}_{m}\left[\mathbb{E}\left[e^{v\theta}|m^{\prime}, m\right] \mathbb{E}\left[e^{v\eta^{\prime}}|m^{\prime}, m\right]|m^{\prime}\right]
\end{aligned}$$

as $\eta^{\prime}$ is independent of θ conditional on m and $m^{\prime}$. Continuing we have

$$Q_{v,m}^{\prime}=\omega \mathbb{E}\left[e^{v\xi^{\prime}}\right]+(1-\omega) \mathbb{E}\left[e^{v\xi^{\prime}}\right] \mathbb{E}_{m}\left[\mathbb{E}\left[e^{v\theta}|m\right] \mathbb{E}\left[e^{v\eta^{\prime}}|m^{\prime}, m\right]|m^{\prime}\right]$$

as θ and m' are independent conditional on m. Note that $\mathbb{E}\left[e^{v\xi'}\right]$ is constant across time and m so we have

$$\begin{aligned} Q'_{v,m} &= \omega\mathbb{E}\left[e^{v\xi'}\right] + (1-\omega)\mathbb{E}_m\left[\mathbb{E}\left[e^{v(\theta+\xi)}|m\right]\mathbb{E}\left[e^{v\eta'}|m',m\right]|m'\right] \\ &= \omega\mathbb{E}\left[e^{v\xi'}\right] + (1-\omega)\mathbb{E}_m\left[\mathbb{E}\left[e^{vy}|m\right]\mathbb{E}\left[e^{v\eta'}|m',m\right]|m'\right] \\ &= \omega\mathbb{E}\left[e^{v\xi'}\right] + (1-\omega)\mathbb{E}_m\left[Q_{v,m}\mathbb{E}\left[e^{v\eta'}|m',m\right]|m'\right] \end{aligned} \tag{11}$$

So to compute the evolution of $Q'_{v,m}$ we need to keep track of the current values of $Q_{v,m}$, the probabilities $\Pr(m|m') = \Pr(m,m')/\Pr(m')$, and the innovation moments which are given by

$$\mathbb{E}\left[e^{v\eta'}|m',m\right] = \mathbb{E}_i\left[e^{v\mu_i+\frac{1}{2}v^2\sigma_i^2}|m',m\right]$$

where i indexes the component of the mixture of normals from which η' is drawn. For this last step, note that (a) the distribution of η' depends on the type of labor market transition the household experiences and this can be inferred from m and m', (b) the means μ_i depend on x.

The probabilities $\Pr(m|m') = \Pr(m,m')/\Pr(m')$ depend on the stocks of households in different labor market statuses. For example,

$$\Pr(m = \text{employed}|m' = \text{employed}) = \frac{(1-\zeta')\Pr(m = \text{employed})}{\Pr(m' = \text{employed})}.$$

In the end, we have eight moments that we need to keep track of: $Q_{v,m}$ for two values of v and three values of m and $\Pr(m)$ for two values of m with the third given by the fact that the three sum to one.

B Fitting the income process

This appendix provides additional information on the simulated method of moments procedure used to select the parameters of the idiosyncratic income process.

Step 1 To simulate the model, I need estimates of λ_t^s, λ_t^ℓ and ζ_t. I estimate these from the relationships

$$u_t^s = \zeta_t(1 - u_{t-1}) \tag{12}$$

$$u_t^m = (1 - \lambda_t^s)u_{t-1}^s \tag{13}$$

$$u_t^\ell = (1 - \lambda_t^\ell)(u_{t-1}^m + u_{t-1}^\ell), \tag{14}$$

where u_t is the unemployment rate and u_t^s, u_t^m, and u_t^ℓ are the short-term, medium-term, and long-term unemployment rates measured as those with durations less than 15 weeks, 15 to 26 weeks and 27 or more weeks. I construct the unemployment rates using data from from the St. Louis Fed FRED database according to

$$u_t^s = (\text{UNEMPLOY} - \text{UEMP15OV})/\text{CLF16OV}$$
$$u_t^m = (\text{UEMP15OV} - \text{UEMP27OV})/\text{CLF16OV}$$
$$u_t^\ell = \text{UEMP27OV}/\text{CLF16OV}$$

where the codes such as UNEMPLOY are identifiers from the FRED database. I time average these monthl series to create quarterly data as a first step.

Step 2 Construct the labor market indicators for x_t and the W_t. For x_t I use the ratio of short-term unemployed (u_t^s as constructed above) to the labor force, the same ratio for long-term unemployed (UEMP15OV/CLF16OV), an index of average weekly hours (PRS85006023), and the labor force participation rate (CIVPART). Each time series is HP filtered with a smoothing parameter of 10^5 to remove trends at very low frequencies. I transform the series to have mean zero and unit standard deviation and then express the resulting series in terms of their principal components. Orthogonalizing the series into principal components should not affect the results in theory, but it is helpful for the numerical analysis. These quarterly data cover 1977:I to 2011:IV. Store these in a matrix X^x. For W_t: I use λ_t^s, and wage and salary income (NIPA Table 2.1 line 3) and deflated with the deflator for personal consumption expenditures (PCECTPI). I rescale the quarterly growth rates of wage and salary income so that the implied annual growth rates match the mean one-year change in earnings from

Guvenen et al.. I express these series in principal components and add a constant. Store these series in a three-column matrix X^W.

Step 3 Guess a vector of parameters

$$\Theta \equiv [\phi_1^x, \cdots, \phi_3^x, \phi_1^W, \cdots, \phi_3^W, \phi^{xW}, \sigma_\xi, \mu_2, \mu_3, \sigma_{\eta,1}, \sigma_{\eta,2}, \sigma_{\eta,3}, p_2, p_3],$$

where the ϕ's are factor loadings such that $x_t = X^x \phi^x$ and $W_t = X^W \phi^W + \phi^{xW} x_t$.

Step 4 Calculate $\mu_{1,t}$, $\mu_{2,t}$ and $\mu_{3,t}$ from Equations (1) - (3). The normalization $\bar{\mu}$ is chosen to satisfy $\mathbb{E}[e^\eta] = 1$ and this requires

$$\bar{\mu}_t = -\log\left(p_{1,t}\exp(\sigma_{\eta,1}^2/2) + p_{2,t}\exp(\mu_2 - x_t + \sigma_{\eta,2}^2/2) + p_{3,t}\exp(\mu_3 - x_t + \sigma_{\eta,3}^2/2)\right), \quad (15)$$

where $p_{2,t} \equiv p_2 \zeta_t (1 - u_{t-1})$, $p_{3,t} \equiv p_3(1 - u_{t-1})$, and $p_{1,t} \equiv 1 - p_{2,t} - p_{3,t}$ are the masses of workers drawing from each component of the mixture distribution.

Step 5 Simulate employment, skill, and mortality shocks for a panel of individuals. The employment transition probabilities are the values for λ_t^s, λ_t^ℓ, and ζ_t computed in step 1. I simulate 100,000 individuals from 1977 through 2011. The results are not sensitive to the way the distribution of θ is initialized because the objects of interest are related to earnings changes as opposed to levels. Guvenen et al. (2014) eliminate individuals with earnings below a time-varying threshold ($1,300 in 2005) when they form their sample. In computing the moments from simulated data I impose the same $1,300 requirement on the sample (I use $1,300 rescaled by average income in the model relative to 2005 average income). This has the effect of eliminating full-year unemployed workers and very few others and it has little effect on the results because I am focussed on matching percentiles, which are not sensitive to outliers.

Step 6 Compute the moments: aggregate the quarterly earnings observations to annual observations, take 1-year, 3-year, and 5-year changes in log earnings. I use the following moments for each year and for each of the 1-year, 3-year and 5-year changes: the median, and the 10th and 90th percentiles. I express the 10th and 90th percentiles relative to the median (i.e. $50-10$ and $90-50$). Doing so implies that any differences between the simulated and empirical medians do not change the targets for the widths of the upper and lower tails. The earnings loss after displacement can be computed as $p_2(\mu_2 - \bar{x})$, where $\bar{x}$ is the average of x_t in expansions or recessions as defined by the NBER dates.

Step 7 Compute the objective function: I take the difference between the simulated moments and the empirical moments and express them as squared percentage differences except for the difference in medians, which is expressed as the squared difference relative to the 90th percentile as in $([\text{model median}] - [\text{data median}]/[\text{data 90th percentile}])^2$. I use the moments reported in Table A13 of Guvenen et al. (2014).

Step 8 Adjust the guess in step 3 and repeat to minimize the objective function from step 7.

As an additional check on the calibrated income process, I compute the standard deviations of the income changes and compared those to the results in Guvenen et al. (2014). Figure 7 shows that the simulated standard deviations are only slightly cyclical while those in the data are more or less acyclical. The simulated standard deviations are somewhat below the observed values.

C Equilibrium conditions

Due to the progressive tax system, an individual with skill θ_i has income proportional to $e^{(1-b^y)\theta_i}$. Given the formulation of the unemployment insurance scheme, this proportionality holds even for unemployed wokrers. This scaling along with homothetic preferences and permanent shocks to θ can be exploited to eliminate one state variable. Specifically, use lower case letters to denote household variables relative to $e^{(1-b^y)\theta_i}$:

$$c_i = \frac{C_i}{e^{(1-b^y)\theta_i}}, \qquad a_i = \frac{A_i}{e^{(1-b^y)\theta_i}}, \qquad k_i' = \frac{K_i'}{e^{(1-b^y)\theta_i}}.$$

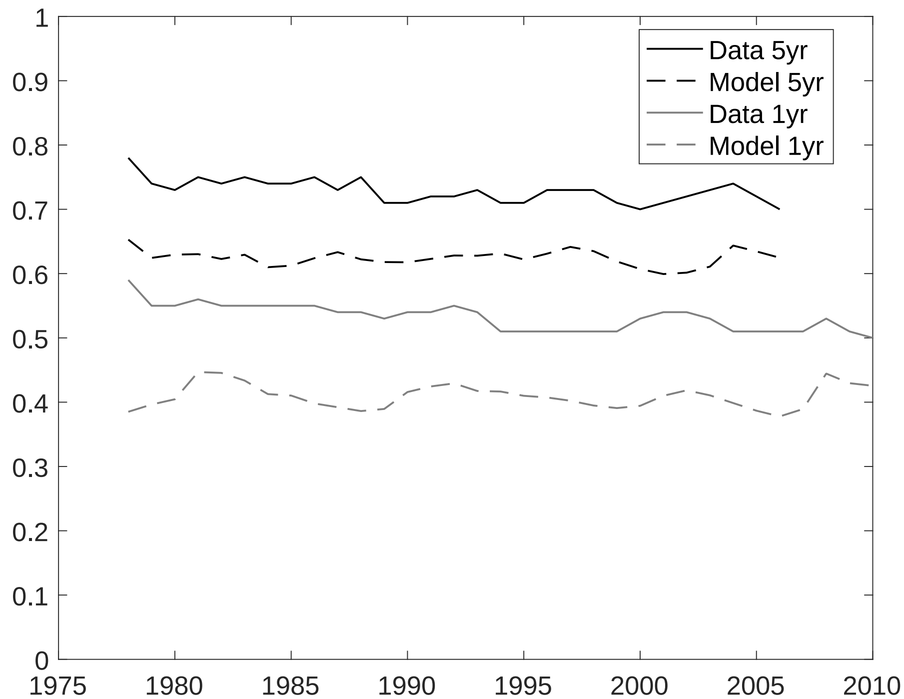

Figure 7: Simulated and empirical standard deviations of the income process.

The Euler equation and budget constraint are

$$C_{i,t}^{-\gamma} \geq \beta_i(1-\omega)\mathbb{E}_t\left[R_{t+1}C_{i,t+1}^{-\gamma}\right]$$
$$C_{i,t} + K_{i,t} = RK_{i,t-1} + (1-\tau)W_t e^{(1-b^y)y_{i,t}} n_{i,t},$$

where recall $n_{i,t} = 1$ for employed, $n_{i,t} = b^{u,s}$ for short-term unemployed, and $n_{i,t} = b^{u,\ell}$ for long-term unemployed. In terms of normalized variables these are

$$c_{i,t}^{-\gamma} \geq \beta_i(1-\omega)\mathbb{E}_t\left[e^{-\gamma(1-b^y)\eta_{i,t+1}}R_{t+1}c_{i,t+1}^{-\gamma}\right] \tag{16}$$
$$c_{i,t} + k_{i,t} = Rk_{i,t-1}e^{-(1-b^y)\eta_{i,t}} + (1-\tau)W_t e^{(1-b^y)\xi_{i,t}} n_{i,t}. \tag{17}$$

The stock of short-term unemployed evolves according to (12) and the stock of long-term unemployed evolves according to

$$u_t^\ell = (1-\lambda_t^s)u_{t-1}^s + (1-\lambda_t^\ell)u_{t-1}^\ell. \tag{18}$$

Budget balance for the social insurance system can be expressed as

$$\mathbb{E}_i\left[n_{i,t}(1-\tau_t)W_t e^{(1-b^y)y_{i,t}}\right] = \mathbb{E}_i\left[I_{n_{i,t}} W_t e^{y_{i,t}}\right]$$

where $\mathbb{E}_i$ is a cross-sectional expectation and the left-hand side is average after tax income and the right-hand side is average pre-tax income, where recall $I_{n_{i,t}}$ indicates employment. We can rewrite this budget constraint as

$$(1-\tau_t)\left[\left(1-u_t^s-u_t^\ell\right)Q_{b,m=1,t} + b^{u,s}u_t^s Q_{b,m=2,t} + b^{u,\ell}u_t^\ell Q_{b,m=3,t}\right] = \left(1-u_t^s-u_t^\ell\right)Q_{1,m=1,t} \tag{19}$$

where $m = 1, 2, 3$ correspond to employed, short-term unemployed, and long-term unemployed, respectively, and the Q moments are defined as in Appendix A.

In addition to the Euler equations and budget constraints, the equations needed to solve the model are: (1), (2), (3), (4), (5), (6), (7), (8), (9), (11) for the six combinations of v and m, (12), (15), (18), and (19). These are 19 equations in the 20 variables $\mu_{1,t}$, $\mu_{2,t}$, $\mu_{3,t}$, $\bar{\mu}_t$, z, u^s, u^ℓ, λ, ζ, x, the six Q's, τ, W, R, and $\bar{K}$. Closing the model requires determining the aggregate capital stock $\bar{K}$. Following Krusell and Smith (1998) this is done in two ways. In solving the household's decision problem, I make use of a forecasting rule

$$\bar{K}' = h(z, \hat{\lambda}, \hat{\zeta}, \hat{u}^s, \hat{u}^\ell, x, \bar{K}, Q). \tag{20}$$

I assume that $h(\cdot)$ is a first-order polynomial. In simulating the model, $\bar{K}$ is determined according to the household decision rules and the dynamics of the distribution of wealth in line with equation (10). To express (10) in normalized terms, note that equations (16) and (17) are independent of the individual's permanent income, $e^{(1-b^y)\theta}$, and so the decision for K' will be proportional to this permanent income. In particular the savings policy rule can be expressed as

$$F(A, \theta, m, S) = e^{(1-b^y)\theta} f(a, m, S).$$

In normalized terms we then have

$$\bar{K}' = \int e^{(1-b^y)\theta} f\left(Ae^{-(1-b^y)\theta}, m, S\right) d\Gamma(A, \theta, m). \tag{21}$$

D Numerical methods

D.1 Method for Section 6

Overview I solve the model using the Krusell-Smith algorithm, which involves solving the household's problem for a given law of motion for the capital stock and updating this law of motion through simulation and least squares curve fitting. For a given law of motion, I solve the household's problem using a projection method on a grid that is constructed from simulated data generated by a guess of the model solution in the manner described by Maliar and Maliar (2015). This requires alternating between solving the decision problem given a grid and simulating the solution and updating the grid. The steps of the algorithm are as follows:

1. Guess household decision rules and a forecasting rule for the aggregate capital stock.
2. Simulate the economy and record aggregate states.
3. Use simulated data to construct a grid for the aggregate state space.
4. Solve the household's decision problem on the grid.
5. Simulate the economy and record aggregate states.
6. Use simulated data to construct a grid for the aggregate state space.
7. If the grid has converged then continue, otherwise return to step 4.
8. Update the forecasting rule with least-squares regression.
9. If the forecasting rule has converged stop, otherwise return to step 4.

Initial guesses A good initial guess is important to the success of this algorithm because a poor guess will lead to a situation in step 5 where the economy is being simulated far from the grid on which the problem was solved. In most cases I have found it sufficient to use the linearized solution for the representative agent model as a starting point. The representative agent's policy rule can be simulated to provide the data for the initial grid and this policy can also serve as a decent guess for the forecasting rule. The success of this guess is premised on the difference between the representative agent and incomplete markets economies being limited. This is not the case for the baseline economy and this guess is not sufficient for this case. Instead, I found it necessary to gradually build up an initial guess based on versions of the model that are more similar to the representative agent model. I gradually lowered the rate of time-preference of the less patient group to generate this guess.

Constructing the grid See Maliar and Maliar (2015). I target a grid with 80 points. I approximate functions of aggregate states with a mix of first- and second-order polynomials. The state vector is

$$S = \{z, \hat{\zeta}, \hat{\lambda}, \bar{K}, \hat{u}^s, \hat{u}^\ell, x, Q_{1,1}, Q_{1,2}, Q_{1,3}, Q_{b,1}, Q_{b,2}, Q_{b,3}\}$$

To approximate unknown functions, I use a complete second order polynomial in $\{z, \hat{\zeta}, \hat{\lambda}, \bar{K}, \hat{u}^s, \hat{u}^\ell, x\}$ augmented with linear terms in the Q variables. In all there is a total of 42 basis functions.[16]

Solving the household's problem I solve the household's problem using the endogenous grid point method (**?**). A household's decision rule can be written in terms of cash on hand relative to permanent income

$$F(A, \theta, m, \beta, S) = e^{(1-b^y)\theta} f(a, m, \beta, S),$$

where $a = Ae^{-(1-b^y)\theta}$. Even though it is not a state, I have included β among the household's states in order to be explicit about the different types of households whose decision rules must

[16]The use of polynomials should not be confused with a perturbation approximation method. The projection method used here minimizes the residual in the model equations across a grid over the state space as opposed to a perturbation method which uses information from the derivatives of the model equations at a single point in the state space.

be solved for. For given values of $m \in \{1,2,3\}$, β, and aggregate state S, I approximate the household's savings function with a piece-wise linear function of 100 knots with more knots placed at low levels of savings and $k'_{[1]} = 0$. I fix a grid on end-of-period savings k' such that

$$k'_{[j]} = e^{(1-b^y)\theta} f\left(a_{[j]}(m,\beta,S), m, \beta, S\right),$$

where $[j]$ indexes grid points and $a_{[j]}(m,\beta,S)$ is the value of normalized cash on hand (relative to $e^{(1-b^y)\theta}$) for which a household with states (m,β,S) saves $k'_{[j]}$. n and β both take two discrete values and there are 100 values of j so the algorithm must find 400 functions that map S to particular values of $a_{[j]}(m,\beta,S)$. I approximate each of these functions as a complete second-order polynomial in S. As there are more grid points than terms in the polynomials approximating these functions, I update the coefficients of the polynomials by least-squares projection.

To compute expectations with respect to aggregate shocks, I use the monomial rule with $2N$ nodes described by Maliar and Maliar (2015). To compute expectations over idiosyncratic shocks I use Gaussian quadrature. Of particular interest is the η shock because this has a time-varying distribution. I use Gaussian quadrature with five points in each tail and three points for the central mixture component. As it is only the means of the distributions that are moving with x and not the variance of the mixture components, I construct fixed quadrature grids for each component and shift their locations according to x. For the transitory shock, ξ, I use Gaussian quadrature with three points.

Simulation and updating the law of motion In solving for the law of motion for the aggregate capital stock, I perform a non-stochastic simulation in the spirit of **?** for 2,200 quarters and discard the first 200 quarters. Using the simulated aggregate capital stock, I update the law of motion with a least squares regression using the same functional form as for the household decision rules (a complete second-order polynomial in the aggregate state).

Accuracy of the law of motion for capital To assess the accuracy of the law of motion for the capital stock, Figure 8 shows a plot of the capital stock generated from simulating the model and the approximate capital stock generated by repeatedly applying the approximate

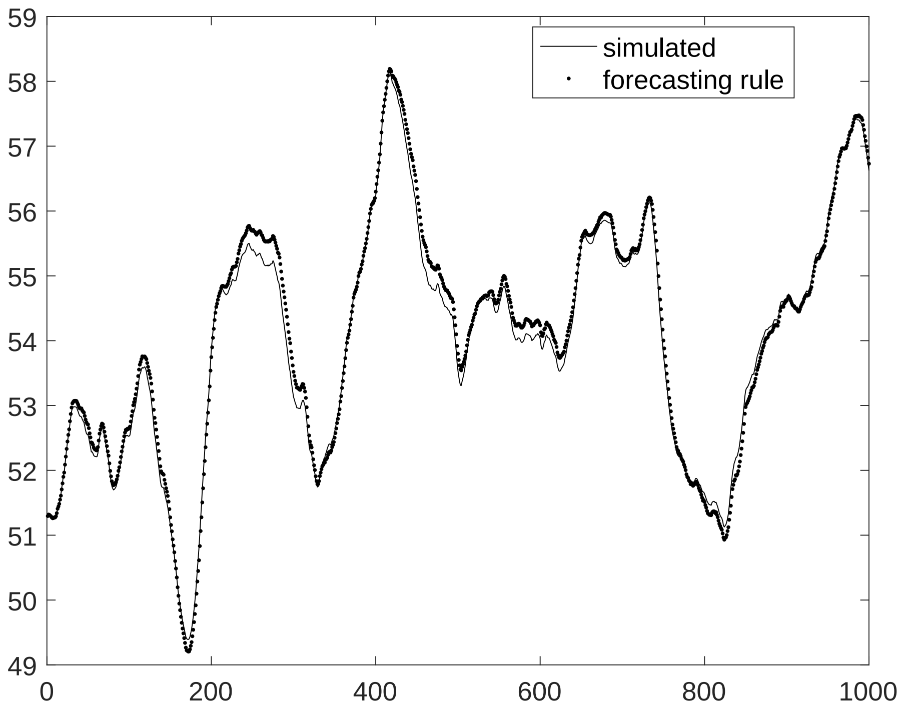

Figure 8: Simulated aggregate capital stock with implied values from $\log K' = h(S)$.

law of motion for capital.[17] This is one sample path of shocks for 1000 quarters and the discrepancy between the two lines is the forecast error that the agents are making at different horizons. One can see that the discrepancy is small even at forecast horizons of 1000 quarters. The maximum absolute log difference between the two series is 0.0087 and the mean absolute log difference is 0.0027. Another commonly-reported accuracy check is the R^2 of the one-step ahead forecast, which is 0.999965.

Accuracy of the policy rules There are several sources of error in the approximate solution. First, there is the error introduced by the discrepancy between the forecasting rule and the actual dynamics of the aggregate capital stock. Second, there are errors associated with the projection method that arise between grid points when the function being approximated is not of the same form as the approximating function.

To assess the accuracy of the solution, I calculate unit-free Euler equation errors.[18] For a given state of the economy, S, the distribution of wealth, the capital stock, and exogenous

[17]As **?** suggests, the sequence of shocks used to simulate the model for the accuracy check differ from those used to calculate the approximate law of motion.

[18]See **?** for an explanation of this accuracy check and the interpretation of the errors in terms of bounded rationality.

variables are predetermined.

$$\text{Pre-determined and exogenous:} \qquad \bar{K}, z, \zeta, \lambda, u^s, u^\ell, x, Q_{1,1}, Q_{1,2}, Q_{1,3}, Q_{b,1}, Q_{b,2}, Q_{b,3}, \Gamma.$$

Γ is generated by a non-stochastic simulation. I then use the computed solution to determine the household decision rules

$$\text{Approx. solutions:} \qquad a_{[j]}(m, \beta, S) \qquad \forall j, m, \beta.$$

Using these policy rules, one can compute the savings of each household and then aggregate to find K' by integrating against Γ. For a given set of aggregate shocks one can then compute S' from (6), (7), (8), (9), (11), (12), and (18). Given S' compute $a_{[j]}(m, \beta, S')$. The Euler equation error is then

$$\frac{\beta(1-\omega)\mathbb{E}\left[e^{-\gamma(1-b^y)\eta'}R(S')c(a', m', \beta, S')^{-\gamma}\right]^{-1/\gamma}}{c(a, m, \beta, S)} - 1$$

where $a' = R(S')k'e^{-(1-b^y)\eta'} + (1-\tau(S'))W(S')e^{(1-b^y)\xi'}[n' + b^u(1-n')]$ and the consumption functions satisfy $c + k' = a$. Here $\mathbb{E}$ represents an expectation over aggregate and idiosyncratic shocks. For aggregate shocks, I use Gaussian quadrature with seven points in each dimension. For idiosyncratic shocks I use the same Gaussian quadrature methods as used to solve the model. For households who are borrowing constrained the Euler equation should not hold. For these households consumption is determined from the borrowing constraint and there is no Euler equation error.

Using these steps, I can compute the Euler equation error for an individual with a particular set of states (aggregate and idiosyncratic). To choose a set of aggregate states at which to evaluate the errors, I simulate the economy for 1000 starting from the risky steady state and repeat this 100 times for different sets of random shocks. This produces 100 points that can be considered as draws from the model's ergodic distribution over the aggregate state space. For idiosyncratic states, I construct a fine grid on normalized cash on hand. Specifically, I use 1000 equally spaced points from 1/1000 to 2000.

Figure 9 summarizes the errors across points in the state space. Each of the panels corresponds to a set of discrete states for an individual. Each panel plots the mean and maximum absolute errors across the 100 aggregate states that were tested.

Solving for the policy rule under complete markets For the complete markets model I use the algorithm described in Maliar and Maliar (2015) that iterates on the Euler equation. I again use a mixture of first- and second-order polynomials for the savings policy rule.[19]

E Complete markets model

This appendix derives the representative agent Euler equation from the environment presented in Section 2 augmented with a complete set of contingent securities. I assume that trade takes place at an initial period prior to date 0 before any uncertainty has been resolved. I also assume that all households have the same rate of time-preference. Like **?**, I assume that all current and future generations meet and trade in this initial period. Let $I_{i,t}$ take the value 1 if household i is alive in period t and zero if it is not. I will treat birth and death as random events against which the household can insure. Specifically, let s^t be a history of stochastic events up to date t the probability of which is $\pi_t(s^t)$. These stochastic events dictate the evolution of all idiosyncratic as well as aggregate developments. Let $p_t(s^t)$ be the date-0 price of a unit of the final good at date t and history s^t. The household's utility function is

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \frac{C_{i,t}^{1-\gamma}}{1-\gamma} I_{i,t}(s^t).$$

Notice that a household only values consumption when it is living. In order to prevent them from choosing negative values of consumption when not living I impose $C_{i,t}(s^t) \geq 0$ for all i, t, and s^t. The household's present-value budget constraint is

$$\sum_{t=0}^{\infty} \sum_{s^t} p_t(s^t) \left[C_{i,t}(s^t) + K_{i,t+1}(s^t) - W_t(s^t)\ell_{i,t}(s^t) - \tilde{R}_t(s^t) K_{i,t}(s^{t-1}) \right],$$

[19] In this case, the variables $Q_{b,1}$, $Q_{b,2}$, and $Q_{b,3}$ are not states because the social insurance system is irrelevant under complete markets. I use a complete second-order polynomial in $\{z, \hat{\zeta}, \hat{\lambda}, \bar{K}, \hat{u}^s, \hat{u}^\ell, x\}$ augmented with linear terms in $Q_{1,1}$, $Q_{1,2}$, and $Q_{1,3}$.

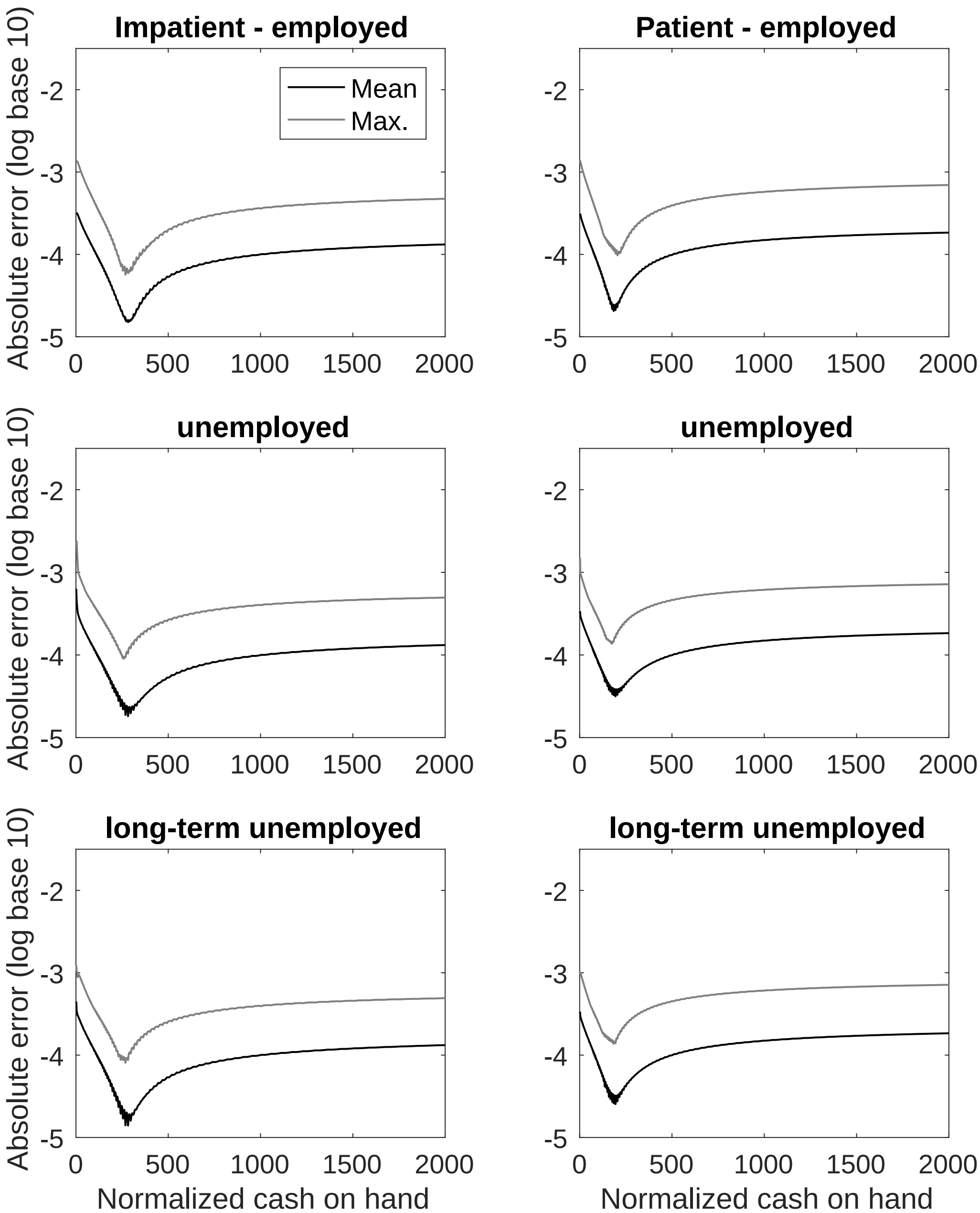

Figure 9: Euler equation errors. Left column: unemployed; right column: employed; top row: less patient; bottom row: more patient. Maximum and mean across 100 aggregate states.

where $\ell_{i,t}$ is the household's endowment of efficiency units of labor. I assume that all households are identical when trade occurs and each is endowed with an equal share of the initial capital stock.

The household's Lagrangian is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \frac{C_{i,t}(s^t)^{1-\gamma}}{1-\gamma} I_{i,t}(s^t) \\ - \Xi_i \sum_{t=0}^{\infty} \sum_{s^t} p_t(s^t) \left[C_{i,t}(s^t) + K_{i,t+1}(s^t) - W_t(s^t)\ell_{i,t}(s^t) - \tilde{R}_t(s^t) K_{i,t}(s^{t-1}) \right] \\ + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \psi_{i,t}(s^t) C_{i,t}(s^t),\end{aligned}$$

where Ξ_i and $\psi_{i,t}(s^t)$ are Lagrange multipliers. The first order condition with respect to consumption is

$$\beta^t \pi_t(s^t) \left[C_{i,t}(s^t)^{-\gamma} I_{i,t}(s^t) + \psi_{i,t}(s^t) \right] = p_t(s^t) \Xi_i.$$

The complementary slackness condition is $\psi_{i,t}(s^t) C_{i,t}(s^t) = 0$. By symmetry, the Lagrange multiplier Ξ_i is common across households. It follows that $C_{i,t}(s^t)^{-\gamma} I_{i,t}(s^t) + \psi_{i,t}(s^t)$ must be common across households at a particular date and history. Suppose the household is living, then consumption is positive and common across living households and $\psi_{i,t}(s^t) = 0$. If the household is not living then $\psi_{i,t}(s^t)$ takes the value of the marginal utility of consumption for living households and consumption is zero. This establishes that all living households consume the same amount regardless of their labor income history or age. Define $\bar{C}_t(s^t)$ as the common level of consumption for living households and note $\bar{C}_t(s^t)^{-\gamma} = C_{i,t}(s^t)^{-\gamma} I_{i,t}(s^t) + \psi_{i,t}(s^t)$.

The first order condition with respect to $K_{i,t+1}(s^t)$ is

$$\Xi_i p_t(s^t) = \sum_{s^{t+1}|s^t} \Xi_i p_{t+1}(s^{t+1}) \tilde{R}_{t+1}(s^{t+1}).$$

Substituting for $\Xi_i p_t(s^t)$ from above yields

$$
\begin{aligned}
& C_{i,t}(s^t)^{-\gamma} I_{i,t}(s^t) + \psi_{i,t}(s^t) \\
&\qquad = \beta \sum_{s^{t+1}|s^t} \frac{\pi_t(s^{t+1})}{\pi_t(s^t)} \left[C_{i,t+1}(s^{t+1})^{-\gamma} I_{i,t+1}(s^{t+1}) + \psi_{i,t+1}(s^{t+1}) \right] \tilde{R}_{t+1}(s^{t+1}) \\
& \bar{C}_t(s^t)^{-\gamma} = \beta \sum_{s^{t+1}|s^t} e^{q_{t+1}(s^{t+1})} \frac{\pi_t(s^{t+1})}{\pi_t(s^t)} \bar{C}_{t+1}(s^{t+1})^{-\gamma} \tilde{R}_{t+1}(s^{t+1}) \\
& \bar{C}^{-\gamma} = \beta \mathbb{E}_t \left[\bar{C}'^{-\gamma} \tilde{R}' \right].
\end{aligned}
$$

F Data definitions for Section 6

Table 3 and Figures 4 and 5 refer to data on aggregate output and consumption. Output is per capita real GDP and consumption is the sum of non-durable and services consumption constructed as

$$
\begin{aligned}
Y &= \log(\text{GDP}) - \log(\text{POP}) - \log(\text{GDPDEF}) \\
C &= \log(\text{PCESV} + \text{PCND}) - \log(\text{POP}) - \log(\text{GDPDEF}),
\end{aligned}
$$

where the codes refer to identifiers from the FRED database

G Additional plots

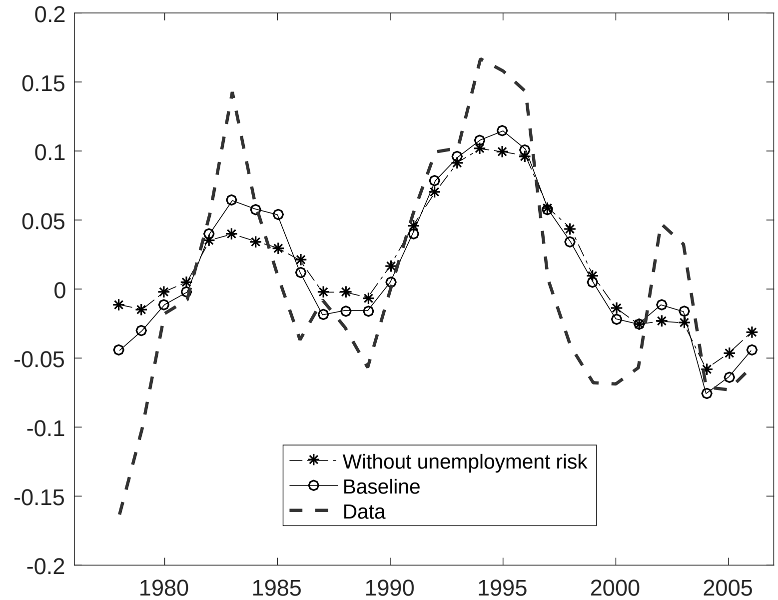

Figure 10: Kelley's skewness of five-year earnings growth rates.

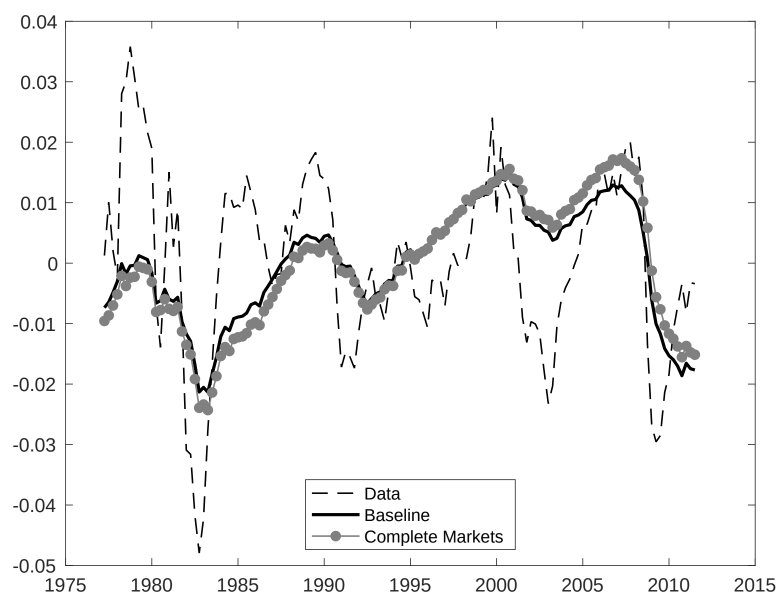

Figure 11: Dynamics of aggregate output implied by labor market shocks. Data refer to per capita GDP deflated with the GDP deflator and detrended with the HP filter with smoothing parameter 1600.

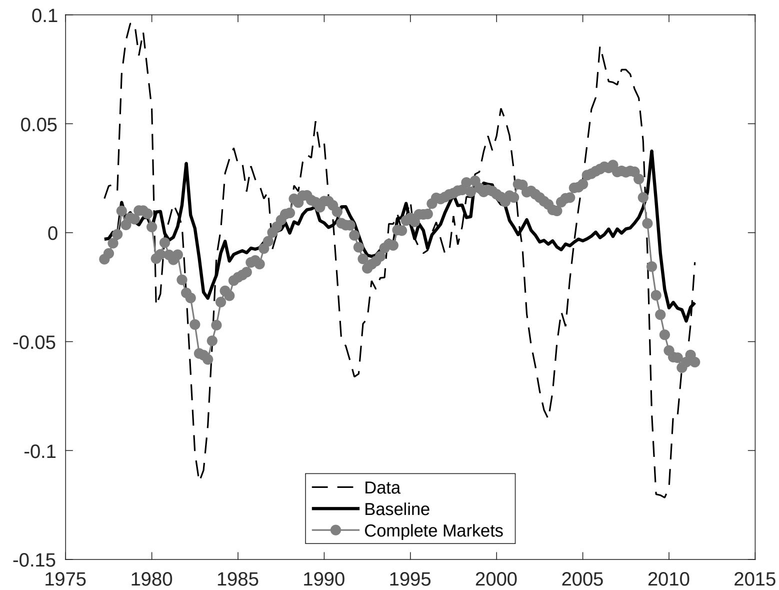

Figure 12: Dynamics of aggregate investment implied by labor market shocks. Data refer to per capita fixed private investment deflated with the GDP deflator and detrended with the HP filter with smoothing parameter 1600.

References

Abowd, J. M. and Card, D. (1989). On the covariance structure of earnings and hours changes. *Econometrica: Journal of the Econometric Society*, pages 411–445.

Arellano, C., Bai, Y., and Kehoe, P. J. (2016). Financial frictions and fluctuations in volatility. Working Paper 22990, National Bureau of Economic Research.

Bachmann, R. and Bayer, C. (2013). Wait-and-see business cycles? *Journal of Monetary Economics*, 60(6):704–719.

Basu, S. and Bundick, B. (2012). Uncertainty shocks in a model of effective demand. Working Paper 18420, National Bureau of Economic Research.

Benabou, R. (2000). Unequal societies: Income distribution and the social contract. *American Economic Review*, pages 96–129.

Blass-Hoffmann, E. and Malacrino, D. (2016). The cyclicality of employment risk. Stanford University Working Paper.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, 77(3):623–685.

Bloom, N. (2014). Fluctuations in uncertainty. *The Journal of Economic Perspectives*, 28(2):153–175.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2012). Really uncertain business cycles. Working Paper 18245, National Bureau of Economic Research.

Carroll, C. D., Slacalek, J., and Tokuoka, K. (2013). Buffer-stock saving in a krusell-smith world. ECB Working Paper.

Castaneda, A., Díaz-Giménez, J., and Ríos-Rull, J.-V. (2003). Accounting for the us earnings and wealth inequality. *Journal of Political Economy*, 111(4):818–857.

Challe, E., Matheron, J., Ragot, X., and Rubio-Ramirez, J. F. (2015). Precautionary saving and aggregate demand. Banque de France Working Paper.

Challe, E. and Ragot, X. (2016). Precautionary saving over the business cycle. *The Economic Journal*, 126(590):135–164.

Coeurdacier, N., Rey, H., and Winant, P. (2015). Financial integration and growth in a risky world. Working Paper 21817, National Bureau of Economic Research.

Constantinides, G. M. and Duffie, D. (1996). Asset pricing with heterogeneous consumers. *Journal of Political economy*, 104(2):219.

Constantinides, G. M. and Ghosh, A. (2014). Asset pricing with countercyclical household consumption risk. Technical report, National Bureau of Economic Research.

Couch, K. A. and Placzek, D. W. (2010). Earnings losses of displaced workers revisited. *American Economic Review*, 100(1):572–89.

Davis, S. J. and von Wachter, T. (2011). Recessions and the costs of job loss. *Brookings Papers on Economic Activity*.

De Santis, M. (2007). Individual consumption risk and the welfare cost of business cycles. *American Economic Review*, 97(4):1488 – 1506.

Diaz-Gimenez, J., Glover, A., and Rios-Rull, J.-V. (2011). Facts on the distributions of earnings, income, and wealth in the united states: 2007 update. *Quarterly Review*.

Feldstein, M. S. (1969). The effects of taxation on risk taking. *Journal of Political Economy*, 77(5):755–764.

Fernández-Villaverde, J., Guerrón-Quintana, P. A., Kuester, K., and Rubio-Ramírez, J. (2015). Fiscal volatility shocks and economic activity. *American Economic Review*, 105(11):3352–3384.

Gilchrist, S., Sim, J. W., and Zakrajšek, E. (2014). Uncertainty, financial frictions, and investment dynamics. Working Paper 20038, National Bureau of Economic Research.

Guvenen, F., Ozkan, S., and Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):621–660.

Heathcote, J., Storesletten, K., and Violante, G. L. (2014). Optimal tax progressivity: An analytical framework. Working Paper 19899, National Bureau of Economic Research.

Hubmer, J. (2016). The job ladder and its implications for earnings risk. Yale University Working Paper.

Huckfeldt, C. (2016). Understanding the scarring effect of recessions. Cornell University Working Paper.

Jacobson, L. S., LaLonde, R. J., and Sullivan, D. G. (1993). Earnings losses of displaced workers. *The American Economic Review*, pages 685–709.

Jarosch, G. (2015). Searching for job security and the consequences of job loss. Stanford University Working Paper.

Krebs, T. (2003). Growth and welfare effects of business cycles in economies with idiosyncratic human capital risk. *Review of Economic Dynamics*, 6(4):846–868.

Krebs, T. (2007). Job displacement risk and the cost of business cycles. *The American economic review*, pages 664–686.

Krueger, D., Mitman, K., and Perri, F. (2015). Macroeconomics and heterogeneity, including inequality. University of Pennsylvania working paper.

Krusell, P. and Smith, Jr., A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Leduc, S. and Liu, Z. (2016). Uncertainty shocks are aggregate demand shocks. *Journal of Monetary Economics*, 82:20 – 35.

MaCurdy, T. E. (1982). The use of time series processes to model the error structure of earnings in a longitudinal data analysis. *Journal of econometrics*, 18(1):83–114.

Maliar, L. and Maliar, S. (2015). Merging simulation and projection approaches to solve high-dimensional problems with an application to a new keynesian model. *Quantitative Economics*, 6(1):1–47.

Perri, F. and Steinberg, J. (2012). Inequality and redistribution during the great recession. Economic Policy Paper 12-1, Federal Reserve Bank of Minneapolis.

Ravn, M. and Sterk, V. (2013). Job uncertainty and deep recessions. UCL Working Paper.

Rothstein, J. and Valletta, R. G. (2014). Scraping by: Income and program participation after the loss of extended unemployment benefits. IZA Discussion Paper No. 8022.

Schmidt, L. (2016). Climbing and falling off the ladder: Asset pricing implications of labor market event risk. University of Chicago Working Paper.

Storesletten, K., Telmer, C. I., and Yaron, A. (2001). The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk. *European Economic Review*, 45(7):1311–1339.

Storesletten, K., Telmer, C. I., and Yaron, A. (2007). Asset pricing with idiosyncratic risk and overlapping generations. *Review of Economic Dynamics*, 10(4):519–548.